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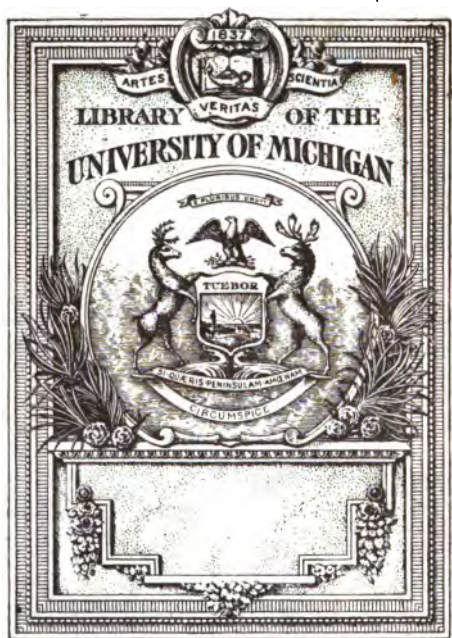
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OR, AN  
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CONTAINING,

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| <p>I. THE Doctrine of <i>Vulgar</i> and <i>Decimal Fractions</i>, wherein the Fundamental Principles are fully and clearly explained in all their Cases.</p> <p>II. THE Extraction of <i>Roots</i>, according to the <i>Newtonian</i> Method, much preferable to</p> | <p>that now taught in Schools.</p> <p>III. THE First Principles of <i>Algebra</i> demonstrated in a very short and easy Method, illustrated with variety of Instances, in the Application thereof to the Solution of Problems.</p> |
|--|--|

For the Use of Schools as well as of Private Gentlemen

Ἐν τῇ Παρισίῳ τῇ ἐκδόσει τυπωθήσεται.

L O N D O N :

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T H E  
D O C T R I N E  
O F

VULGAR FRACTIONS.

D E F I N I T I O N I.



**U**NITY is an abstract Idea of all those Things we call *one*, and is therefore put for 1, and these are either the same or different. I say, Unity, Unit, or One is, by which we distinguish, discern, know, name or express any Thing that is, to be one.

2. Number is a Multitude, or a Many, of Units.

I L L U S T R A T I O N.

As Unit, or One, is that, by which every Thing that is, is expressed to be one; so Number is that by which we express what contains Quantity or Multitude of those Things, Quantities, or Magnitudes we desire to have named, known or signified: As two Men, three Books, six Things, nine Months, forty Pounds of Money, sixty hundred Weight, two thousand Ells, and so *in infinitum*. If the Number be referred to, or compared with a given Unity, it is called a *determined Number*; but if referred to an indefinite Unity, it's an *undetermined Number, or Quantity*; as for the *Length* of a Bowl-

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a Bowling-green, if that is to be determined, we must first assume some Quantity at Pleasure for Unity, and find the Relation it hath thereto; thus the *Length* will be expressed differently, according to the different Unity assumed.

3. Part is one Number of a greater of the same Kind, when the lesser measures the greater Number.

### ILLUSTRATION.

One Number is said to measure the greater of the same Kind, when the lesser is many times contained in the greater exactly; as 2 doth measure 6, for 2 is contained in 6, 3 times exactly; 4 doth measure 32, for 4 is contained in 32, 8 times, therefore 4 is the one eighth Part of 32; and so on *in infinitum*. Hence it is manifestly evident, that Unit measures all Numbers, and is a Part of any Number; as of 8, it is the one eighth, of 20 it is the one twentieth Part, &c.

4. Parts is one Number of a greater of the same Kind, when the lesser Number doth not measure the greater.

### ILLUSTRATION.

When the lesser Number doth not measure the greater of the same Kind, then we must conceive that the Unit doth measure the same greater Number, and consequently a Part thereof, and therefore, that the lesser Number is so many such Parts of the greater, as the same lesser Number doth contain Units. As 7 doth not measure 15, but the Unit doth measure them both, for of the former it is one seventh, and of the latter it is the one fifteenth; likewise 9 doth not measure 47, but Unit measures them both, for of the former Part it is the one ninth, and of the latter one forty seventh Part, and therefore 7 is seven fifteenth Parts of 15, and 9 is nine forty seventh Parts of 47, and so *in infinitum*.

5. Many times is one Number of a lesser of the same Kind, when the lesser Number doth measure the greater.

### ILLUSTRATION.



ILLUSTRATION.

This is the Converse of the third Definition ; for as the lesser Number is such a Part of a greater Number of the same Kind, so the greater Number is so many times of the lesser Number of the same Kind ; as 9 is measured by 3 at 3 times, consequently 9 is 3 times of 3.

6. Things may be said to be equal, when one may be substituted in the Room of the other, without changing the Quantity ; and tho' two Things be unequal, a Part of the greater may be substituted for the Whole of the other, or the lesser may be substituted for the Part of the other, without changing the Quantity.

7. An *aliquot* Part is that which being repeated a certain Number of Times becomes equal to the Whole, but when it becomes either more or less, it is an *aliquant* Part ; therefore a lesser Number is an *aliquot* Part of a greater, when the lesser Number measures the greater Number exactly, and, by *Definition the 2d of the 5th Book of Euclid*, the greater is then called a *Multiple* of the lesser, that is, a Magnitude of a Magnitude, the Greater of the Lesser. Thus 6 is an *aliquot* Part of 30, and therefore by this Definition 30 is a Multiple of 6, for 6 times 5 is 30 ; but 8 is an *aliquant* Part of 30, because 8 does not measure 30, for 3 times 8 is 24, and therefore deficient of 30. Generally one Number is said to be *multiple* to another, when it contains it a certain Number of Times without a Remainder, and the Number contained is the *submultiple*.

8. Quantities are *commensurable* when there is an *aliquot* Part common to both, or when one is an *aliquot* Part of the other, otherwise they are called *incommensurable*.

9. Quantities are *homogeneous*, when one of them taken a certain Number of Times will exceed the other, or subtracted will leave nothing, or a Number less than itself ; but when one of them repeated cannot exceed the other, they are *heterogeneous*.

10. Numbers are either abstract, as 4, 5, 6, or concrete,

## 4 *The Doctrine of Vulgar Fractions.*

crete, when we determine what they are; and if they are referred to the same Unity, they are called *homogeneous* Numbers, but if to different, they are *heterogeneous*; as 5 Globes of Gold, and 10 of Silver. Therefore a whole Number may be abstract, that is, having no Denomination to it, or concrete; but a Fraction may wholly be concrete, that is, having a Denomination annex'd to it, or partly abstract, or partly concrete. For *three fifths*, considered as a Fraction, is *abstract*, since it is not expressed what it is the *three fifths* of, but in respect of its Denomination *fifths*, it is *concrete*; also three fifth Parts of a Yard is *concrete*, both in respect of its Denomination *fifths*, and in respect of what they are fifths of.

11. A prime Number is that which can only be measured by Unity, as 5, 7, 11.

12. A composite Number is that which other Numbers besides Unity will measure, as 6 measures 12 by 2, and 2 measures 12 by 6.

13. A common Measure is a Number by which two or more Numbers are measured; as 6 measures 12 and 30, being twice in the one, and 5 times in the other.

14. Numbers that are prime to one another, are such as have no other common Measure but an Unit.

15. An Integer or whole Number is referred to Unity, as the Whole to a Part, but a broken Number as a Part to the Whole, and is expressed by 2 Numbers, called a Vulgar Fraction, one above, which is the Numerator, and the lower the Denominator; for the Whole is broken into a certain Number of Parts, and the upper Number is so many of those Parts; as  $\frac{4}{5}$ , *four fifths*. Therefore it shews that the Integer is divided into 5 Parts, and the Line above exhibits that 4 of those 5 Parts are taken or expressed, and therefore 4 is called the Numerator, and 5 its Denominator, because it denotes the Name, or Denomination thereof, and generally

$$\frac{N}{D} = \frac{\text{Numerator}}{\text{Denominator}}$$

16. A rational Number is that which can be measured by

by Unity ; an irrational cannot, and so is called a Surd, or inexpressible.

17. A whole Number is that which numbereth Wholes.

18. A broken Number, or Fraction, numbereth the Parts of an Unit.

19. A mixt Number is made by putting a Fraction to a whole Number. .

20. A perfect Number is that which is equal to all its own aliquot Parts taken together.

21. A diminutive Number is greater than the Sum of all its aliquot Parts.

22. An abounding Number is less than the Sum of all its aliquot Parts.

23. An abstract Number is that which hath no Denomination annexed to it.

24. A concrete Number is such as hath a Denomination annexed to it.

25. A single Fraction is such as denotes a Part or Parts of some Whole.

26. A compound Fraction is such as denotes a Part or Parts of some Part or Parts.

27. A proper Fraction is less than an Unit.

28. An improper Fraction is greater than an Unit.

29. Ratio is, that by which is considered, known, named and expressed what one Quantity is, of another of the same Kind ; as that it is greater, lesser or equal. But, more precisely, Ratio is the Consideration and Knowledge of the Quantity or Number, by which it is exactly expressed and named, what Quantity one Quantity is, of another of the same Kind ; therefore Ratio is the Relation of Things homogeneous, which determines the Quantity of one from the Quantity of the other, without using another homogeneous ; the Things compared are called the Terms of the Ratio, that which is referred to the other being the Antecedent, and that to which it is referred, the Consequent.

## C O R O L L A R Y I.

Seeing in Fractions the Relation of the Numerator to

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the Denominator is understood without assuming a third homogeneous, the same will express a Ratio.

### C O R O L L A R Y II.

If two Quantities are compared together without assuming a third, either they are equal or not, that is, it is either a Ratio of Equality or Inequality.

### C O R O L L A R Y III.

If the Terms of the Ratio be unequal, either the lesser is referred to the greater, or the greater to the lesser; the former as a Part to the Whole, the latter as the Whole to the Part. The Ratio will thus determine how often the lesser is contained in the greater, or how often the greater contains the lesser, that is, how much of the greater the lesser is equal to, which Division discovers.

### C O R O L L A R Y IV.

A Ratio being in itself intelligible, we may thereby discern the Relation of Things, tho' they are not present to Sense, or compared with one another.

30. A Ratio is said to be rational, which is as 1, or as a rational Number to another, as 2 to 3; but it's irrational, when it cannot be expressed by rational Numbers. *Example.* Let there be two Quantities, as A and B, and let A be less than B; if you take A from B as often as it can be taken, suppose 7 times, there will remain either something or nothing; in the latter Case A will be to B as 1 to 7, or A will be equal to  $\frac{1}{7}$  B: Thus it is rational. In the former Case, either there will be a Part, which being taken, suppose 4 times from A, and 9 times from B, leaves nothing, or there will be no such Part of the former, then A will be to B as 4 to 9, or A equal  $\frac{4}{9}$  B, and so the Ratio is rational; but if the latter, the Ratio of A to B cannot be expressed in rational Numbers; that is, it cannot be told what Part A is to B.

31. The Exponent of the Ratio is the Quotient that ariseth

aristeth from dividing the Antecedent by the Consequent.  
*Example.* Exponent of the Ratio of 3 to 2 is  $1\frac{1}{2}$ , but of 2 to 3 is  $\frac{2}{3}$ .

### COROLLARY I.

If the Consequent be 1, the Antecedent is the Exponent of the Ratio, as the Exponent of the Ratio of 2 to 1 is 2, of 4 to 1 is 4. Thus an Integer or whole Number expresses the Ratio of many to one, or of a Multitude to Unity.

### COROLLARY II.

The Exponent of the Ratio is to Unity, as the Antecedent to the Consequent.

### COROLLARY III.

Ratios are understood by their Exponents, and therefore if the Antecedent be A, and the Consequent B, the Ratio of A to B may be expressed thus,  $\frac{A}{B}$ .

32. If the lesser Term be an *aliquot* Part of the greater, the Ratio of greater Inequality is called *multiple*, but the Ratio of lesser Inequality *sub-multiple*; particularly, if in the first Case the Exponent be 2, the Ratio is double, if 3, triple, &c. In the other Case, if the Exponent be  $\frac{1}{2}$ , the Ratio is sub-double, if  $\frac{1}{3}$  sub-triple; as 2 to 6 is a sub-triple Ratio, but 6 to 2 is a triple Ratio.

### COROLLARY.

If A be to B as C to D, then the Identity of the Ratio may be expressed thus,  $\frac{A}{B} = \frac{C}{D}$ , or thus,  $A : B :: C : D$ .

33. The Similitude or Identity of two Ratios is called Proportion, and the Quantities that have the same Ratio are said to be proportional. *Example.* If A be to B as C to D, then these 4 Numbers are proportional.

34. Proportion is Similitude, or Likeness of Ratios, and doth consist at the least in three Terms.

## ILLUSTRATION.

Proportion is Similitude, or Likeness; of *Ratios*, between the Quantities compared; as of 4 to 2, and 10 to 5, the *Ratio* is 2 times, or double, in both Comparisons. Therefore Proportion doth consist at the least in three Terms, and that is, when the Consequent is taken as an Antecedent to the next Consequent, as 18 to 6, and 6 to 2; the *Ratio* is 3 times, or triple in both Comparisons, and so *in infinitum*.

## T H E O R E M I.

**I**F a Number cannot be divided by some prime Number, nor greater than the Square Root thereof, that Number is a Prime.

## T H E O R E M II.

**I**F two Fractions are equal to one another, the Numerator of the one is in such Proportion to its Denominator, as the Numerator of the other to its Denominator.

## T H E O R E M III.

**I**F the Numerator of a Fraction be less than the Denominator, that Fraction is a proper one. But if the Numerator be equal or greater than the Denominator, the Fraction is improper.

## T H E O R E M IV.

**E**VERY whole Number may be expressed in the Form of a Vulgar Fraction, without altering its Value, if that whole Number be the Numerator, and Unity the Denominator.

## T H E O R E M V.

**Q**uantities that have the same Ratio that one rational Number hath to another, are commensurable.

DEMON-

# DEMONSTRATION.

An Unit is an aliquot Part of a rational Integer, and a Fraction hath an aliquot Part in common with an Unit; therefore such as are to one another, as one rational Number to another, one of them is either an aliquot Part of the other, or else there is one aliquot Part common to both, therefore they are commensurable.

## THEOREM VI.

**C**OMMENSURABLE Quantities among themselves are either as an Unit to a rational whole Number, or as a rational whole Number is to another; but it is not so with Incommensurables.

# DEMONSTRATION.

In commensurable Quantities, either one is an aliquot Part of the other, or else there is an aliquot Part common to both; and if in the former Case, the lesser Quantity be taken for Unity, and in the latter a common aliquot Part, in the first Case it will answer the greater Quantity, and in the second Case the rational whole Number will answer both; therefore, in the first Case the Quantities are among themselves as Unity; and in the second as a rational whole Number to another, which was the first Part.

In Incommensurables there is no common aliquot Part (*as by Definition 8.*) therefore there is no Unity to which they can be commensurable; wherefore seeing every rational Number is commensurable with Unity, they cannot be as one rational Number to another, which was the second Part.

## THEOREM VII.

**T**WO Quantities multiplying each other, have the same Product.

# DEMONSTRATION.

Let the two Factors be A and B, then 1 to A, B will be

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be to A B; and 1, to B, A will be to B A; therefore A B is equal B A. *Example*,  $7 \times 5 = 5 \times 7 = 35$ .

### C O R O L L A R Y.

Let there be three Factors A, B and C; because  $A B = B A$ , then  $C A B = C B A$ , and so  $A B C = B A C$ ; and because  $B C = C B$ , then  $A C B = A B C$ , and so  $C B A = B C A$ ; wherefore  $C A B = C B A = A B C = B A C = A C B = B C A$ , that is, there is the same Product in whatever Order they are multiplied.

### T H E O R E M VIII.

**A**LL Products are composite Numbers.  
This is self-evident, and needs no Demonstration.

### T H E O R E M IX.

**I**F the Numerator be equal to the Denominator, as  $\frac{5}{5}$ , the Fraction is equivalent to an Integer; if less, it is less than an Integer; if more, it is greater.

### D E M O N S T R A T I O N.

For the Denominator shews the Whole broken into so many Parts, and the Numerator is so many of those Parts (*by Definition 14.*); if then they be equal, the Fraction is equal to the very Integer itself; as  $\frac{3}{3}$  of a Groat is a Groat itself; if less, then there are some Parts taken, but not all, and so it is less; if more, then there are more Parts taken than the Integer hath, and so the Integer is equal to a Part of the Fraction, and so greater.

### S C H O L I U M.

Those that are equal or greater than the Integer are spurious, or improper, Fractions.

### T H E O R E M X.

**I**F the Consequents of two Ratios be equal, they are as the Antecedents; but if the Antecedents be equal, they are reciprocally as the Consequents.

DEMON-



DEMONSTRATION.

For  $\frac{7}{7} : \frac{9}{7} : 7 : 9$

And  $\frac{1}{9} : \frac{1}{7} : 7 : 9$ .

*Characters used in the following Sheets, for the more convenient operating the Rules, are as follows.*

- + IS a Sign, denoting Addition; as if  $5+7+9$ , &c. then it would be the same as 5 added to 7, and that Sum added to 9, &c.
- Is a Sign, denoting Subtraction; as  $7-5$ , shews that you must subtract 5 from 7.
- x Is a Sign, denoting Multiplication; as  $5 \times 7 \times 9$ , &c. shews that 5 is to be multiplied by 7, and that Product again by 9, &c.
- ÷ Is a Sign, denoting Division, as  $8 \div 4$  shews that 8 is to be divided by 4. But in the following Sheets I more particularly use Division thus  $\frac{8}{4}$ , the top Figure always denoting the Dividend, and the bottom the Divisor; and it evidently follows that when the top Number is less than the bottom, then it is self-evident that it must stand so, and from thence are derived all Vulgar Fractions. *Example.* If  $\frac{8}{4}$ , then the Quotient will be 2, which I set thus,  $\frac{8}{4} = 2$ ; also  $\frac{56}{4} = 14$ .
- But if  $\frac{11}{17}$ , then it cannot be divided by 17, therefore it must so stand, and consequently a Vulgar Fraction. I also denote Division thus  $4)8(2$ , and  $4)56(=14$ ; and so the rest.
- = A Sign, denoting Equality, as  $4+2=6$ , that is, 4 and 2 is equal to 6.

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:: A proportional Sign, as if 2 Yards cost 5 Pounds, 4 Yards will cost 10 Pounds, which I set thus, 2 : 5 :: 4 : 10.

✓ Is a Sign denoting the Square-Root.

∛ or  $\sqrt[3]{\quad}$  Is a Sign, denoting the Cube-Root, and

∜ or  $\sqrt[4]{\quad}$  the Biquadratic Root.

Having treated of the Definitions, Theorems, with their Demonstrations, &c. being the Foundation for such a necessary Work, it is now time we should come to explain the Subject itself, as it is a very just Observation, that Arts are more easily attained by Examples than Precepts.





O F

# VULGAR FRACTIONS.



*ANY* whole Thing, or an Unit, may be considered as divided into any Number of equal Parts, which have their Name from the Number contained in that Unit.

As if Unit be conceived to be divided into five equal Parts, those Parts are called *fifths*, and are thus written  $\frac{1}{5}$ .

And any Number of those Parts, suppose *three*, are thus expressed, *three fifths*, or three divided by five, and thus written  $\frac{3}{5}$ , and called a Fraction.

## COROLLARY I.

Whence it follows, that a *Fraction* is but a *Quotient*, signifying a *Part* or *Parts* of an Unit, expressed by a *Numerator* as a *Dividend*, and a *Denominator* as a *Divisor* (See Definition 15, also page 11.)

Thus *one seventh* Part of any Thing is the *Quotient* of that Thing divided by *seven*, which by *common Division* is expressed thus,  $\frac{1}{7}$ .

Also  $\frac{3}{4}$  signifies *three fourth* Parts of an Unit, or *one fourth* Part of *three Units*.

## COROLLARY II.

As the Numerator is to the Denominator,  
So is the Fraction to Unit.

For the Dividend is to the Divisor,  
As the Quotient is to Unit.

Thus  $4 : 5 :: \frac{4}{5} : 1$ .

C

COROL-

# 14. Of VULGAR FRACTIONS.

## COROLLARY III.

*Fractions* having the same *Denominators* are one to another as their *Numerators*.

Thus  $\frac{3}{7} : \frac{4}{7} :: 3 : 4$

For  $3 : \frac{3}{7} :: 7 : 1$

And  $4 : \frac{4}{7} :: 7 : 1$  } by preced. Cor.

Theref.  $\frac{3}{7} : \frac{4}{7} :: 3 : 4$

## COROLLARY IV.

As any Fraction is to Unit,

So is Unit to the reverse Fraction.

Thus  $\frac{4}{7} : 1 :: 1 : \frac{7}{4}$

For  $\frac{4}{7} : 1 :: 4 : 7$  } by Cor. 2.

And  $1 : \frac{7}{4} :: 4 : 7$

Theref.  $\frac{4}{7} : 1 :: 1 : \frac{7}{4}$

## SCHOLIUM.

Since it often happens, in *reducing*, *adding*, *subtracting*, &c. *Fractions*, that they swell into too great Numbers, which are more troublesome, and not so manageable as smaller ones, therefore we shall in the next Place shew the way of *reducing* them into their *least Terms*; either before or after such Operations, as there is occasion; which is done by the help of the following

## PROBLEM.

**T**WO Numbers being given, to find their greatest common Measure, (i. e.) the greatest Number that can divide both without remainder.

## RULE.

Divide the greatest by the least, and that Divisor by the Remainder continually, till nothing remain, and the last Divisor will be the greatest common Measure.

# Of VULGAR FRACTIONS.

45

## EXAMPLE I.

If the Numbers be 91 and 117, or  $\frac{21}{117}$ .  
The greatest common Measure is 13. For

OPERATION.

$$\begin{array}{r} 91 \overline{)117}(1 \\ \underline{91} \\ 26 \overline{)91}(3 \\ \underline{78} \\ 13 \overline{)26}(2 \\ \underline{26} \end{array}$$

Or thus

$$\begin{array}{r} 117 \overline{)1} \\ 91 \overline{)3} \\ 26 \overline{)2} \\ 13 \overline{)1} \end{array}$$

## EXAMPLE II.

If the Numbers be 468 and 846, or  $\frac{468}{846}$ .  
The greatest common Measure is 18; for

OPERATION.

$$\begin{array}{r} 468 \overline{)846}(1 \\ \underline{468} \\ 378 \overline{)468}(1 \\ \underline{378} \\ 90 \overline{)378}(4 \\ \underline{360} \\ 18 \overline{)90}(5 \\ \underline{90} \end{array}$$

Or thus

$$\begin{array}{r} 846 \overline{)1} \\ 468 \overline{)1} \\ 378 \overline{)4} \\ 90 \overline{)5} \\ 18 \overline{)1} \end{array}$$

## EXAMPLE III.

If the Numbers be 5808 and 10080, or  $\frac{5808}{10080}$ .  
The greatest common Measure is 48.

## OPERATION.

$$5808 \overline{) 10080} (1$$

$$\underline{5808}$$

$$4272 \overline{) 5808} (1$$

$$\underline{4272}$$

$$1536 \overline{) 4272} (2$$

$$\underline{3072}$$

$$1200 \overline{) 1536} (1$$

$$\underline{1200}$$

$$336 \overline{) 1200} (3$$

$$\underline{1008}$$

$$192 \overline{) 336} (1$$

$$\underline{192}$$

$$144 \overline{) 192} (1$$

$$\underline{144}$$

$$48 \overline{) 144} (3$$

$$\underline{144}$$

..

Or thus

$$10080 \overline{) 1}$$

$$5808 \overline{) 1}$$

$$4272 \overline{) 2}$$

$$1536 \overline{) 1}$$

$$1200 \overline{) 3}$$

$$336 \overline{) 1}$$

$$192 \overline{) 1}$$

$$144 \overline{) 3}$$

$$48 \overline{) 1}$$

## EXAMPLE IV.

What is the greatest common Measure of 46198 and 871624?

## OPERATION.

Or thus

46198)871624(18	871624 18
<u>46198</u>	46198 1
409644	40060 6
<u>369584</u>	6138 1
40060)46198(1	3232 1
<u>40060</u>	2906 8
6138)40060(6	326 1
<u>36828</u>	298 10
3232)6138(1	28 1
<u>3232</u>	18 1
2906)3232(1	10 1
<u>2906</u>	8 4
	2
	326)2906(8
	<u>2608</u>
	298)326(1
	298
	<u>28</u>
28)298(10	
<u>280</u>	
18)28(1	
<u>18</u>	
10)18(1	
<u>10</u>	
8)10(1	
<u>8</u>	
2)8(4	
<u>8</u>	
	<u>..</u>

*Note. If 1 be the greatest common Measure, the Numbers are said to be Prime to one another.*

## PROPOSITION I.

**G**IVEN two or more Numbers, to find a common Multiple to them.

## R U L E.

*Multiply continually all the given Numbers, the Product is the Number sought.*

## E X A M P L E.

Find a common Multiple to 2, 4, 5, 6 and 8.

*Answer.* Thus  $2 \times 4 = 8$ ,  $8 \times 5 = 40$ ,  $40 \times 6 = 240$ ,  $240 \times 8 = 1920$ .

## P R O P O S I T I O N II.

**G**IVEN two Numbers, to find their least common Multiple.

## R U L E.

*Find first their greatest common Measure, then divide by that Measure either of the Numbers, and the Quotient multiplied by the other of those Numbers, the Product will be the Number sought.*

## E X A M P L E.

What is the least common Multiple of 16 and 24.

Thus  $16 \overline{) 24} ( 1$

greatest C. M.  $= 8 \overline{) 16} ( 2 \quad 8 \overline{) 24} ( 3 \times 16 = 48$ . the *Ans.*

## P R O P O S I T I O N III.

**T**O reduce a Fraction into its least Terms.

## R U L E.

*Divide the Numerator and Denominator by their greatest common Measure, their Quotients will be a Fraction equivalent to the former, and in the least Terms.*

## E X A M P L E I.

If the given Fraction be  $\frac{91}{117}$ .

The greatest common Measure is 13.

And  $13 \overline{) 91} ( 7$  the given Fraction in its least Terms.

$13 \overline{) 117} ( 9$ .

E x.



## EXAMPLE II.

If the given *Fraction* be  $\frac{468}{846}$ .

The *greatest common Measure* is 18.

And  $\begin{array}{r} 18 \overline{) 468} 26 \\ 18 \overline{) 846} 47 \end{array}$  the given *Fraction* in the least Terms.

*Note.* When the *greatest common Measure* is 1, the *Fraction* is already in the *smallest Terms*.

## COROLLARY.

A *Fraction*, whose Terms are even, may be abbreviated by a *continual Division* by 2.

Thus  $\frac{384}{512} = \frac{3}{4}$ .

For  $\begin{array}{r} 2 \overline{) 384} \mid \frac{192}{256} \mid \frac{96}{128} \mid \frac{48}{64} \mid \frac{24}{32} \mid \frac{12}{16} \mid \frac{6}{8} \mid \frac{3}{4} \end{array}$

You may also abbreviate a *Fraction* to its lowest Term, if you will observe this

## RULE.

*Divide the Numerator and Denominator (if they be both even) by 2, 4, 6, 8, &c. If the Numerator and Denominator be one even and the other odd, or both odd, try some odd Number, as 3, 5, 7, 9, &c. repeat this Division as often as you can, so shall the last Quotient of the Denominator be a new Numerator, and the last Quate of the Denominator, a new Denominator.*

## EXAMPLE I.

Reduce  $\frac{45360}{73600}$  into its lowest Terms.

## OPERATION.

$\begin{array}{r} 12 \quad 9 \quad 7 \quad 5 \quad 4 \\ 45360 \mid 3780 \mid 420 \mid 60 \mid 12 \mid 3, \text{ so that } \frac{3}{4} = \frac{45360}{73600} \end{array}$

## EXAMPLE II.

Reduce  $\frac{216}{432}$  into its lowest Terms.

## OPERATION.

$\begin{array}{r} 4 \quad 2 \quad 3 \quad 9 \\ 216 \mid 54 \mid 27 \mid 9 \mid 1, \text{ so that } \frac{1}{2} = \frac{216}{432} \end{array}$

But

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But the best and general Way of reducing a Fraction to its lowest Terms, is by the greatest common Measure, which we have exemplified above.

### COROLLARY.

When both Terms have Cyphers adjoining cut off equal Cyphers from both.

Thus  $\frac{1}{4} \frac{0000}{0000} = \frac{1}{4}$ , for  $\frac{1}{4} \frac{0000}{0000} = \frac{1}{4}$ .

And  $\frac{1}{4} \frac{00}{00} = \frac{1}{4}$ , for  $\frac{1}{4} \frac{00}{00} = \frac{1}{4} = \frac{1}{4}$ .

### PROPOSITION IV.

**T**O reduce an Integer into an improper Fraction.

#### CASE I.

*Where there is no Denominator assigned.*

#### RULE.

*Let the given Integer be a Numerator, and Unit its Denominator.*

Thus  $2 = \frac{2}{1}$ ;  $11 = \frac{11}{1}$ ,  $17 = \frac{17}{1}$ ,  $57 = \frac{57}{1}$ .

*For dividing by Unit does not diminish the Value.*

#### CASE II.

*Where there is a Denominator assigned.*

#### RULE.

*Multiply the Integer by the assigned Denominator, the Product shall be the Numerator.*

#### EXAMPLE I.

To reduce 13 into an improper Fraction, whose Denominator shall be 12.

Thus  $\frac{13 \times 12}{12} = \frac{156}{12}$ , then  $\frac{156}{12}$  is the required Fraction.

#### EXAMPLE II.

Reduce 39 into an improper Fraction, whose Denominator shall be 29.

Thus

Thus  $\frac{39 \times 29}{29} = \frac{1131}{29}$ , then  $\frac{1131}{29}$  is the required Fraction.

## EXAMPLE III.

Reduce 3821 into an improper Fraction, whose Denominator shall be 684.

Thus  $\frac{3821 \times 684}{684} = \frac{2613564}{684}$ , then  $\frac{2613564}{684}$  is the required Fraction.

For  $2613564 : 684 :: 3821 : 1$ .

And  $2613564 : 684 :: \frac{2613564}{684} : 1$ .

Therefore  $3821 = \frac{2613564}{684}$ .

Or  $3821 = \frac{3821}{1} = \frac{3821 \times 684}{684 \times 1} = \frac{3821 \times 684}{684} = \frac{2613564}{684}$ .

## PROPOSITION V.

**T**O reduce mixt Fractions into improper ones.

### RULE.

*Multiply the Integer by the Denominator of the Fraction, and add the Numerator to the Product, subscribing the same Denominator.*

### EXAMPLE I.

Reduce  $5\frac{3}{4}$  into an improper Fraction.

Thus  $\frac{5 \times 4 + 3}{4} = \frac{20 + 3}{4} = \frac{23}{4}$  the required Fraction.

### EXAMPLE II.

Reduce  $47\frac{2}{5}$ , into an improper Fraction.

Thus  $\frac{47 \times 5 + 2}{5} = \frac{235 + 2}{5} = \frac{237}{5}$  the required Fraction.

### EXAMPLE III.

Reduce  $36\frac{3}{4}$  into an improper Fraction.

Thus  $36\frac{3}{4} = \frac{36 \times 4 + 3}{4} = \frac{144 + 3}{4} = \frac{147}{4}$ .

For

For the *Unit* here is considered as divided into 4 *equal* Parts.

Therefore the *Integers* must be multiplied by 4 to produce 4ths.

To which 3 *fourths* being added, the sum will be also *fourths*.

### PROPOSITION VI.

**T**O reduce an *improper Fraction* into an *Integer*, or *mixt Fraction*.

#### RULE.

*Divide the Numerator by the Denominator, and the Quotient will be the Integer, or mixt Fraction required.*

#### EXAMPLE I.

Reduce  $\frac{156}{12}$  into its whole or mixt Number.

Thus  $\frac{156}{12} = 13$ . For 156 divided by 12 = 13.

### DEMONSTRATION.

For  $\frac{156}{12} : 1 :: 156 : 12$ .

And  $13 : 1 :: 156 : 12$ .

Therefore  $\frac{156}{12} = 13$ .

#### EXAMPLE II.

Reduce  $\frac{147}{4}$  into its whole or mixt Number,

Thus  $\frac{147}{4} = 36\frac{3}{4}$ .

#### EXAMPLE III.

Reduce  $\frac{25576}{130}$  to its whole or mixt Number.

Thus  $\frac{25576}{130} = 188\frac{1}{17}$ .

### PROPOSITION VII.

**T**O reduce a *Fraction* into its *Equivalent*, that shall have any assigned Denominator.

#### RULE.

*Multiply the Numerator of the Fraction by the assigned Denominator, and divide the Product by the Denominator* of

# Of VULGARS FRACTION. 23

*of the Fraction; the Quotient shall be the Numerator required.*

## EXAMPLE I.

To reduce  $\frac{3}{4}$  into a Fraction, whose Denominator is 12.

Thus  $\frac{3 \times 12}{4} = \frac{36}{4} = 9$ , and  $\frac{36}{9} = 4$ , then  $\frac{36}{4}$  is the required

Fraction, or  $\frac{3}{4} = \frac{4)3 \times 12}{12} = \frac{4)36}{12} = \frac{9}{3}$

For  $4 : 3 :: 12 : \frac{12 \times 3}{4} = \frac{36}{4} = 9$

Therefore  $\frac{3}{4} (= \frac{4)3 \times 12}{12}) = \frac{9}{3}$

## EXAMPLE II.

To reduce  $\frac{11}{12}$  into a Fraction whose Denominator is 84.

Thus  $\frac{11 \times 84}{12} = \frac{924}{12}$  and  $\frac{924}{12} = 77$ , then  $\frac{77}{12}$  is the re-

quired Fraction; or, much neater,

Thus  $\frac{11}{12} = \frac{12)11 \times 84}{84} = \frac{12)924}{84} = \frac{77}{7}$

## EXAMPLE III.

To reduce  $\frac{7}{8}$  into a Fraction whose Denominator is 56.

Thus  $\frac{7}{8} = \frac{8)7 \times 56}{56} = \frac{8)392}{56} = \frac{49}{8}$  the required Fraction.

## COROLLARY.

By this *Proposition*, *Fractions* are reduced into their known Parts of *Time*, *Measure*, *Weight*, *Coin*, &c. As also into *Decimals*, *Sexagesimals*, &c. and the contrary.

Thus  $\frac{3}{4} \text{ £.} = (\frac{3 \times 20}{4} =) 15 \text{ s.}$

And  $\frac{2}{3} \text{ Deg.} = (\frac{2 \times 60}{3} =) 40'.$

Also  $\frac{7}{11} \text{ Deg.} = 38', 10'', 54''', 32''''', \&c.$

For 1st.  $\frac{7}{11}$  Deg.  $= (\frac{7 \times 60}{11} =) 38' \frac{2}{11}$ .

2d.  $\frac{2}{11}$  Min.  $= (\frac{2 \times 60}{11} =) 10'' \frac{10}{11}$ .

3d.  $\frac{10}{11}$  Sec.  $= (\frac{10 \times 60}{11} =) 54''' \frac{6}{11}$ .

4th.  $\frac{6}{11}$  Thirds  $= (\frac{6 \times 60}{11} =) 32'''' \frac{8}{11}$ , &c.

## S C H O L I U M.

Hence also, to reduce a *Fraction* into its *Equivalent*, that shall have any *assigned Numerator*.

## EXAMPLE I.

To reduce  $\frac{3}{4}$  into a *Fraction*, whose *Numerator* is 9.

Thus  $\frac{3}{4} = (\frac{9}{(3)4 \times 9} = \frac{9}{(3)36} =) \frac{9}{12}$ .

For  $3 : 4 :: 9 : \frac{9 \times 4}{3} = \frac{36}{3} = 12$ .

*This Corollary is in Mr Jones's excellent Treatise, intitled Synopsis Palmari-  
orum Mathematicos, p. 93.*

## EXAMPLE II.

To reduce  $\frac{7}{8}$  into a *Fraction*, whose *Numerator* is 21.

Thus  $\frac{7}{8} = (\frac{21}{(7)8 \times 21} = \frac{21}{(7)168} =) \frac{21}{144}$  the required *Frac-*

*tion.*

## PROPOSITION VIII.

**T**O reduce *Fractions of different Denominators into their Equivalents, which shall have the same Denominator.*

## R U L E.

*Multiply all the Denominators continually, for a common Denominator, and each Numerator continually by the other's Denominators for Numerators, that is, all the Denominators multiplied continually together for a new Denominator, and each Numerator by all the Denominators, except its own; and the Products are the respective Numerators.*

Ex-

## EXAMPLE I.

Reduce  $\frac{2}{3}$  and  $\frac{3}{4}$  into a common Denominator.

### OPERATION.

Thus  $\frac{2}{3} = \frac{8}{12}$ , and  $\frac{3}{4} = \frac{9}{12}$ .

For  $3 \times 4 = 12$  the Denominator common }  
And  $2 \times 4 = 8$  a Numerator. }  $\frac{8}{12}$ .

Likewise  $3 \times 4 = 12$  the common Denom. as before }  
 $3 \times 3 = 9$  a Numerator, therefore }  $\frac{9}{12}$ .

## EXAMPLE II.

$\frac{4}{3}, \frac{5}{6}, \frac{7}{8}$  reduced make  $\frac{192}{1440}, \frac{200}{1440}, \frac{210}{1440}$ .

For  $\begin{cases} 5 \times 6 = 30 \times 8 = 240 \text{ Denominator common.} \\ 4 \times 6 = 24 \times 8 = 192, \text{ a Numerator, or } \frac{192}{1440}. \\ 5 \times 5 = 25 \times 8 = 200, \text{ a Numerator, or } \frac{200}{1440}. \\ 7 \times 5 = 35 \times 6 = 210, \text{ a Numerator, or } \frac{210}{1440}. \end{cases}$

## EXAMPLE III.

$\frac{1}{2}, \frac{3}{4}, \frac{5}{7}$ , make  $\frac{28}{36}, \frac{42}{36}, \frac{40}{36}$ , or  $\frac{14}{18}, \frac{21}{28}, \frac{20}{28}$ .

For  $\begin{cases} \frac{1}{2} = \frac{1 \times 4 \times 7}{2 \times 4 \times 7} = \frac{28}{56} = \frac{14}{28} \\ \frac{3}{4} = \frac{3 \times 2 \times 7}{4 \times 2 \times 7} = \frac{42}{56} = \frac{21}{28} \\ \frac{5}{7} = \frac{5 \times 2 \times 4}{7 \times 2 \times 4} = \frac{40}{56} = \frac{20}{28} \end{cases}$   $\frac{14}{28}, \frac{21}{28}$  and  $\frac{20}{28}$  are the required Fractions.

## EXAMPLE IV.

$\frac{3}{5}, \frac{5}{7}$  and  $\frac{8}{11}$  make  $\frac{231}{385}, \frac{275}{385}$  and  $\frac{280}{385}$ .

For  $\begin{cases} \frac{3}{5} = \frac{3 \times 7 \times 11}{5 \times 7 \times 11} = \frac{231}{385} \\ \frac{5}{7} = \frac{5 \times 5 \times 11}{7 \times 5 \times 11} = \frac{275}{385} \\ \frac{8}{11} = \frac{8 \times 5 \times 7}{11 \times 5 \times 7} = \frac{280}{385} \end{cases}$   $\frac{231}{385}, \frac{275}{385}$ , and  $\frac{280}{385}$  are the respective Fractions.

## EXAMPLE V.

$\frac{3}{5}, \frac{5}{11}, \frac{6}{13}, \frac{11}{21}, \frac{17}{55}$  make  $\frac{495+95}{825825}, \frac{375375}{825825}, \frac{38150}{825825},$   
 $\frac{432575}{825825}, \frac{255255}{825825}.$

For	{	$\frac{3}{5} = \frac{3 \times 11 \times 13 \times 21 \times 55}{5 \times 11 \times 13 \times 21 \times 55} = \frac{495495}{825825}.$	} their respective Fractions.
		$\frac{5}{11} = \frac{5 \times 5 \times 13 \times 21 \times 55}{11 \times 5 \times 13 \times 21 \times 55} = \frac{375375}{825825}$	
		$\frac{6}{13} = \frac{6 \times 5 \times 11 \times 21 \times 55}{13 \times 5 \times 11 \times 21 \times 55} = \frac{38150}{825825}$	
		$\frac{11}{21} = \frac{11 \times 5 \times 11 \times 13 \times 55}{21 \times 5 \times 11 \times 13 \times 55} = \frac{432575}{825825}$	
		$\frac{17}{55} = \frac{17 \times 5 \times 11 \times 13 \times 21}{55 \times 5 \times 11 \times 13 \times 21} = \frac{255255}{825825}$	

## S C H O L I U M.

Hence, to find *two Integers*, that shall be one to the other as two given *Fractions*.

-Suppose the *Fractions* were  $\frac{2}{3}$  and  $\frac{5}{7}$ .

Then  $\frac{2}{3} : \frac{5}{7} :: \frac{14}{21} : \frac{15}{21}$  by this Proposition.

But  $\frac{14}{21} : \frac{15}{21} :: 14 : 15$ , by Cor. 3. page 14.

Therefore  $\frac{2}{3} : \frac{5}{7} :: 14 : 15$ .

## PROPOSITION IX.

**T**O reduce compound Fractions into single ones.

## R U L E.

*Multiply the Numerators continually for a new Numerator, and multiply the Denominators continually for a new Denominator.*

## EXAMPLE I.

Reduce  $\frac{2}{3}$  of  $\frac{3}{4}$  into a single Fraction.

Thus  $2 \times 3 = 6$  the Numerator }  $\frac{6}{12} = \frac{1}{2}$  the required  
 And  $3 \times 4 = 12$  the Denominator } Fraction.

## EXAMPLE II.

Reduce  $\frac{2}{3}$  of  $\frac{4}{7}$  into a single Fraction.

Thus



Thus  $\frac{2 \times 4}{3 \times 7} = \frac{8}{21}$ .

For if  $\frac{2}{7}$  be  $= A$ , then  $\frac{2}{3}$  of  $\frac{4}{7}$  is  $\frac{2}{3} A = \frac{2}{3} \times A$ .

Therefore  $\frac{2}{3}$  of  $\frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$ .

### EXAMPLE III.

Reduce  $\frac{2}{3}$  of  $\frac{4}{9}$  of  $\frac{5}{11}$  of  $\frac{16}{21}$  of  $\frac{81}{95}$  into a single Fraction.

Thus  $\frac{2 \times 4 \times 5 \times 16 \times 81}{5 \times 9 \times 11 \times 21 \times 95} = \frac{51840}{987325} = \frac{1920}{36575}$ .

### PROPOSITION X.

**T**O reduce a Fraction from one Denominator to another.  
And this is either Ascending or Descending. I. A-  
scending.

#### EXAMPLE I.

Reduce  $\frac{4}{3}$  of a Penny to a Fraction of a Pound.

Thus  $\frac{1}{2}$  of  $\frac{1}{12}$  of  $\frac{1}{20}$ . For  $1d.$  is the  $\frac{1}{12}$  of  $\frac{1}{20}$ .

Therefore  $\frac{4}{3}$  is the  $\frac{4}{3}$  of  $\frac{1}{12}$  of  $\frac{1}{20}$ .

Then  $\frac{4 \times 1 \times 1}{5 \times 12 \times 20} = \frac{4}{1200} = \frac{1}{300}$  the required Fraction.

#### EXAMPLE II.

Reduce  $\frac{13}{49}$  of a Penny into the Fraction of a Pound Sterling.

For the same Reasons as above, that  $1d.$  is the  $\frac{1}{12}$  of  $\frac{1}{20}$  of a Pound Sterling.

So is  $\frac{13}{49}$  of  $\frac{1}{12}$  of  $\frac{1}{20}$ .

Then  $\frac{13 \times 1 \times 1}{49 \times 12 \times 20} = \frac{13}{11760}$  the required Fraction.

#### EXAMPLE III.

Reduce  $\frac{1}{7}$  of an Ounce *Averdupois Weight* into the Fraction of an Hundred Weight.

Thus  $\frac{1}{7}$  of  $\frac{1}{16}$  of  $\frac{1}{112}$  of a Hundred.

Then  $\frac{1 \times 1 \times 1}{7 \times 16 \times 112} = \frac{1}{12344}$ , the required Fraction.

EXAMPLE IV.

Reduce  $\frac{5}{6}$  of a Penny Weight *Troy* into the Fraction of a Pound *Troy*.

Thus  $\frac{5}{6}$  of  $\frac{1}{20}$  of  $\frac{1}{12}$  of a Pound.

$$\text{Then } \frac{5 \times 1 \times 1}{9 \times 20 \times 12} = \frac{5}{2160} = \frac{1}{432}.$$

SCHOLIUM.

2. Hence we may reduce a descending Fraction.

EXAMPLE I.

Reduce  $\frac{4}{1280}$  of a Pound into the Fraction of a Penny.

Thus  $4 \times 20 \times 12 = 960$ .

And  $\frac{960}{1280} = \frac{3}{4}$  the required Fraction.

EXAMPLE II.

Reduce  $\frac{13}{1760}$  of a Pound into a Fraction of a Penny.

Thus  $13 \times 20 \times 12 = 3120$ .

And  $\frac{3120}{1760} = \frac{13}{49}$  the required Fraction. As is manifestly evident, being the converse of *Example 2.* of this Proposition.

PROPOSITION XL.

**T**O find the Value of a Vulgar Fraction.

R U L E.

*Multiply the Numerator by the Parts of the next inferior Denomination, and divide the Product by the Denominator, the Quotient gives the Value in the Parts you multiplied by; and if after this Division any Thing remain, multiply that Remainder by the next inferior Denomination, dividing the Product by the Denominator as before; and so proceed till you can bring it no lower, and if then any Thing remain, set it for a Numerator over the former Denominator.*

EXAMPLE I.

What is the Value of  $\frac{6}{8}$  of a Pound Sterling?

Thus

Thus  $\frac{6 \times 20}{8} = \frac{120}{8} = 15s.$  the required Value.

EXAMPLE II.

What is the  $\frac{17}{19}$  of a Pound Sterling.

Thus  $\frac{17 \times 20}{19} = \frac{340}{19}.$

And  $19)340(17$  *Shillings*

19

150

133

17 Remainder

12 next inferior Denomination,

$19)204(10$  *Pence*

19

14

4. next (or lowest) inferior Denomination

$19)56(2$  *Farthings.*

38

18

So  $\frac{17}{19}l. = 17s. 10d. \frac{1}{2} \frac{18}{19}.$

EXAMPLE III.

What is the Value of  $\frac{3}{5}$  of a Shilling.

Thus

3

12 Pence

$5)36(7d. \frac{1}{5} \text{ or } \frac{3}{5}s. = 7d. \frac{1}{5}.$

35

1

EXAMPLE IV.

What is the Value of  $\frac{7}{9}$  of a Penny?

Thus

7

4

$9)28(3far. \frac{1}{9}.$

27

1

so  $\frac{7}{9}d. = \frac{3}{4}d. \frac{1}{9}.$

## EXAMPLE V.

What is the Value of  $\frac{2}{3}$  of a Pound *Troy*?

Thus

$$\begin{array}{r} 2 \\ 12 \\ 9 \overline{)24}(2 \\ 18 \\ \hline 6 \\ 20 \\ 9 \overline{)120}(13 \\ 117 \\ \hline 3 \\ 24 \\ 9 \overline{)72}(8 \\ 72 \end{array}$$

So that  $\frac{2}{3}$  lb. *Troy* = 2oz. 13pw. 8gr.

## EXAMPLE VI.

What is the  $\frac{1}{8}$  of 11C. 3qr. 16lb. *Averdupois* Weight?

Thus

C. qr. lb.

11 3 16

$$\begin{array}{r} 4 \\ 47 \\ 28 \\ \hline 382 \\ 95 \\ \hline 1332 \\ 18 \end{array}$$

10656

1332

lb.

C.

qr.

lb.

29)23976(826 $\frac{22}{3}$ =7 1 14 $\frac{21}{3}$ , the required  
22 Remainder. [Value

## EXAMPLE VII.

What is the Value of  $\frac{1}{8}$  of 13l. 18s. 6d.  $\frac{1}{2}$ ?

Thus

	l.	s.	d.
Thus	13	18	$6\frac{1}{2}$
			<u>5</u>
	6)69	12	$8\frac{1}{2}$
	<u>£. 11</u>	<u>12</u>	<u><math>1\frac{1}{4}\frac{2}{3}</math></u>

## EXAMPLE VIII.

What is the  $\frac{1}{2}$  of 19*C.* 2*qr.* 11*lb.*

	<i>C.</i>	<i>qr.</i>	<i>lb.</i>
Thus	19	2	11
			<u>11</u>

12)215	2	9	
	<u>17</u>	<u>3</u>	<u><math>24\frac{1}{2}</math></u>

the required Value

## EXAMPLE IX.

What Part of a Pound Sterling is  $\frac{2}{3}$  of  $\frac{5}{9}$ ?

First  $\frac{2}{3}$  of  $\frac{5}{9} = \frac{10}{27}$ . For  $\frac{2 \times 5}{3 \times 9} = \frac{10}{27}$  by *Proposition 9*.

Therefore  $\frac{2}{3}$  of  $\frac{5}{9} = \frac{10}{27}$ .

## EXAMPLE X.

What is the  $\frac{1}{4}$  of 13*d.*  $\frac{1}{2}$ ? By observing the Operation as in the above it is 8*d.* 0*q.*  $\frac{6}{8}$ . I think from what I have here delivered, it will be easy for any one to find the Value of any Fraction; and the Reason of my dwelling so much on this last Proposition, is, I have found it to be of good Use to Beginners.

## PROPOSITION •XII.

**T**O find of what Whole any proposed contract Number is any Part or Parts?

### R U L E.

*Suppose the Fraction inverted, and then work as in the last Proposition.*

## EXAMPLE I.

What is 15*s.* the  $\frac{1}{4}$  of?

Thus

## Of VULGAR FRACTIONS.

Thus  $\frac{15 \times 4}{3} = \frac{60}{3} = 20s.$

So 15s. is the  $\frac{3}{4}$  of 20s.

## EXAMPLE II.

What is 17s. 6d. the  $\frac{7}{8}$  of?

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 17 \quad 6 \\ \hline 8. \\ \hline 7 \overline{) 7 \quad 00 \quad 0} \\ \underline{7 \quad 00 \quad 0} \\ \text{£.} \quad 1 \quad 00 \quad 0 \end{array}$$

So that 17s. 6d. is the  $\frac{7}{8}$  of 1l.

## EXAMPLE III.

What is 3C. 2q. 5lb. 13oz.  $\frac{1}{3}$  the  $\frac{1}{4}$  of?

Thus 3 C. 2 q. 5 lb. 13  $\frac{1}{2}$  oz. 6 for  $\frac{1}{3} \times \frac{6}{1} = \frac{6}{3} = 2.$

$$\begin{array}{r} 21 \quad 1 \quad 7 \quad 0 \\ \hline 4 \\ \hline 11 \overline{) 85 \quad 1 \quad 0 \quad 0} \\ \underline{7 \quad 3 \quad 0 \quad 0} \quad \text{the Answer.} \end{array}$$

## EXAMPLE IV.

What is the 3q. 1lb. 5oz.  $\frac{7}{8}$  the  $\frac{16}{21}$  of?

Thus 3 q. 1 lb. 5  $\frac{7}{8}$  oz.

$$\begin{array}{r} 7 \\ \hline 5 \quad 1 \quad 9 \quad 5 \quad \frac{7}{8} \\ \hline 3 \\ \hline 16 \overline{) 16 \quad 0 \quad 0 \quad 0} \\ \underline{1 \quad 0 \quad 0 \quad 0} \end{array}$$

So that 3q. 1lb. 5oz.  $\frac{7}{8}$  is the  $\frac{16}{21}$  of a Hundred Weight.

## PROPOSITION XIII.

**T**O find what Part or Parts any contract Number is of any other of the same Kind.

## R U L E.

*Let both the Numbers be reduced to the least Name found, and make the lesser the Numerator, and the greater the Denominator of a Fraction, which so done reduce it to its least Name.*

## EXAMPLE I.

What Part of 4*l.* 6*s.* 8*d.* is 2*l.* 3*s.* 4*d.*?

Thus  $\begin{array}{ccc} \text{l.} & \text{s.} & \text{d.} \\ 2 & 3 & 4 \end{array} = \frac{520}{1040}$  and  $\frac{520}{1040} = \frac{1}{2}$ .

And  $\begin{array}{ccc} 4 & 6 & 8 \end{array} = \frac{1040}{1040}$   
Therefore 2 3 4 is  $\frac{1}{2}$  of 4*l.* 6*s.* 8*d.* as is self-evident.

## EXAMPLE II.

What Part of 11*l.* 11*s.* 10*d.* is 6*l.* 13*s.* 8*d.*?

Thus  $\begin{array}{ccc} \text{l.} & \text{s.} & \text{d.} \\ 11 & 11 & 10 \end{array} = \frac{2782}{1391}$

And  $\begin{array}{ccc} 6 & 13 & 8 \end{array} = \frac{1604}{1391}$

Therefore  $\frac{802}{1391}$  is the Answer.

## COROLLARY.

Hence might the Parts be found in Measure, Weights, Quantities, Time, &c.



# A D D I T I O N.

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## P R O P O S I T I O N XIV.

**I**F the Denominators are not equal, they must be reduced to such as have equal ones ; then by this

### R U L E.

*The Sum of the Numerators set over the common Denominator shall be the Sum of the given Fractions.*

## D E M O N S T R A T I O N.

As the Denominators contain the Number of Units the Numerators have, (by *Definition 15, pag. 4.*) so the Numerators are only to be added ; and because that cannot be done, except they be homogeneous, (by *Definition 9, pag. 3.*) so they are brought to the same Denominator.

### C A S E I. E X A M P L E I.

Add  $\frac{2}{3}$  and  $\frac{1}{3}$ .

Thus  $2 \times 3 = 6$  the Denominator.

$2 \times 1 = 2$  a Numerator.

$3 \times 1 = 3$  a Numerator.

Then  $\frac{2+3}{6} = \frac{5}{6}$  the Sum.

### E X A M P L E II.

Add  $\frac{3}{4}$  to  $\frac{1}{5}$ .

Thus  $4 \times 5 = 20$  Denominator.

$3 \times 5 = 15$  Numerator.

$4 \times 1 = 4$  Numerator.

Then  $\frac{15+4}{20} = \frac{19}{20} = 1\frac{1}{20}$  the Sum.

### E X A M P L E III.

Add  $\frac{2}{7}$ ,  $\frac{5}{11}$  and  $\frac{1}{3}$ .

Thus



Thus  $7 \times 11 \times 31 = 2387$  a common Denominator.

$2 \times 11 \times 31 = 682$  a Numerator.

$5 \times 7 \times 31 = 1085$  a Numerator.

$19 \times 7 \times 11 = 1463$  a Numerator.

Therefore  $\frac{682 + 1085 + 1463}{2387} = \frac{3230}{2387} = 1\frac{443}{2387}$  the required Sum.

EXAMPLE IV.

Add  $\frac{1}{4}\frac{1}{3}$  to  $\frac{3}{3}\frac{1}{2}\frac{9}{8}$ .

Thus  $415 \times 5326 = 2210290$  Denominator.

$415 \times 3119 = 1294385$  a Numerator.

$113 \times 5326 = 601838$  a Numerator.

Therefore  $\frac{601838 + 1294385}{2210290} = \frac{1896223}{2210290}$  the required Sum.

CASE II. EXAMPLE V.

Add 13 to  $21\frac{1}{3}$ .

Thus  $13 + 21\frac{1}{3} = 34\frac{1}{3}$  the required Sum.

EXAMPLE VI.

Add 544 to  $261\frac{34}{37}\frac{128}{119}$ .

Thus  $544 + 261\frac{34}{37}\frac{128}{119} = 805\frac{34}{37}\frac{128}{119}$  the required Sum.

CASE III. EXAMPLE VII.

Add  $\frac{3}{11}$  to 47.

Thus  $47 + \frac{3}{11} = 47\frac{3}{11}$  the Sum required.

EXAMPLE VIII.

Add  $\frac{4}{8}\frac{1}{8}$  to 429.

Thus  $429 + \frac{1}{2} = 429\frac{1}{2} = 429\frac{4}{8}\frac{1}{8}$  the required Sum.

CASE IV. EXAMPLE IX.

Add  $6\frac{1}{2}$  to  $34\frac{1}{3}$ .

Thus  $6 + 34 = 40$ .

And  $\frac{1}{2} + \frac{1}{3} = \frac{6+5}{10} = 1\frac{1}{6} = 1\frac{5}{30}$ .

Therefore  $40 + 1\frac{5}{30} = 41\frac{5}{30}$  the required Sum.

## EXAMPLE X.

Add  $27\frac{7}{9}$  to  $31\frac{11}{11}$ .

Thus  $27 + 31 = 58$ .

And  $\frac{7}{9} + \frac{11}{11} = \frac{99 + 147}{198} = \frac{246}{198} = 1\frac{57}{99}$ , by *Case I*.

Therefore  $58 + 1\frac{57}{99} = 59\frac{57}{99}$  the Sum required.

## EXAMPLE XI.

Add  $28\frac{2}{7}$ ,  $59\frac{5}{11}$  and  $64\frac{12}{11}$ .

Thus  $28 + 59 + 64 = 151$ .

And  $\frac{2}{7} + \frac{5}{11} + \frac{12}{11} = 1\frac{443}{887}$ , by *Ex. 3. Case I*.

Therefore  $151 + 1\frac{443}{887} = 152\frac{443}{887}$  the required Sum.

## EXAMPLE XII.

Add  $871\frac{3}{5}$   
 $94\frac{5}{11}$   
 $4621\frac{6}{13}$   
 $\frac{11}{21}$   
 $432\frac{17}{33}$

Thus  $871 + 94 + 4621 + 432 = 6018$ .

And  $\frac{3}{5} + \frac{5}{11} + \frac{6}{13} + \frac{11}{21} + \frac{17}{33}$  reduced to a common Denominator, by *Proposition 8, Ex. 5*: is  $\frac{495495}{823823} + \frac{375375}{823823} + \frac{381150}{823823} + \frac{432575}{823823} + \frac{255255}{823823} = \frac{1939850}{823823} = 2\frac{288200}{823823}$  and  $2\frac{288200}{823823} + 6018 = 6020\frac{288200}{823823}$  the required Sum.

I have insisted the longer on this *Case*, because it is most in Practice, and some Care to be had in the Reduction, which I would advise the Reader to be well acquainted with before he proceed to Subtraction; he will then find no Difficulty, but what he will easily surmount.

## CASE V. EXAMPLE XIII.

Add  $\frac{3}{11}$ ,  $\frac{4}{11}$ ,  $\frac{5}{11}$ ,  $\frac{6}{11}$ ,  $\frac{7}{11}$ ,  $\frac{9}{11}$ .

This requires only the Numerators to be added, for the Denominators are all a-like, and to the Sum of the Numerators subscribe the Denominator.

Thus  $\frac{3+4+5+6+7+9}{11} = \frac{34}{11} = 3\frac{1}{11}$  the required

Sum.

Ex-

EXAMPLE XIV.

Add  $\frac{18}{131}$ ,  $\frac{21}{131}$ ,  $\frac{53}{131}$ ,  $\frac{116}{131}$ .

Thus  $\frac{18+21+53+116}{131} = \frac{208}{131} = 1\frac{77}{131}$ .

EXAMPLE XV.

Add  $\frac{2}{9}$  to  $\frac{5}{9}$ .

Then  $\frac{5}{9} + \frac{2}{9} = \frac{5+2}{9} = \frac{7}{9}$ .

For  $\frac{5}{9} : \frac{2}{9} :: 5 : 2$ .

And  $\frac{5}{9} + \frac{2}{9} : \frac{2}{9} :: 5+2 : 2$ .

But  $\frac{5+2}{9} : \frac{2}{9} :: 5+2 : 2$ .

Therefore  $\frac{5}{9} + \frac{2}{9} : \frac{2}{9} :: \frac{5+2}{9} : \frac{2}{9}$ .

Consequently  $\frac{5}{9} + \frac{2}{9} = \frac{5+2}{9} = \frac{7}{9}$ .

This is the *Demonstration* why there must be a common Denominator before Addition can be had.

CASE VI. EXAMPLE XVI.

Add  $\frac{14}{27}$  to  $\frac{3}{4}$  of  $\frac{5}{8}$ .

First  $\frac{3}{4}$  of  $\frac{5}{8}$  must be reduced to a single Fraction, by *Proposition 9*. and it is  $\frac{15}{32} = \frac{5}{8}$ .

Then  $\frac{14}{27} + \frac{5}{8} = \frac{112+135}{216} = \frac{247}{216} = 1\frac{31}{216}$ , by *Ex. I*.

*Case I*

EXAMPLE XVII.

Add  $\frac{131}{251}$  to  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{5}{8}$ .

First  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{5}{8} = \frac{30}{72}$ , by *Prop. 9*.

And  $\frac{131}{251} + \frac{30}{72} = \frac{7530+9432}{18072} = \frac{16962}{18072}$ , by *Case I*.

CASE VII. EXAMPLE XVIII.

Add  $\frac{4}{3}$  of a Pound to  $\frac{7}{8}$  of a Shilling.

E

Thus

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Thus  $\frac{4}{5}$  of a Pound is  $=\frac{4}{5}$  of  $\frac{1}{20}$ .

And  $\frac{4 \times 20}{1 \times 5} = \frac{80}{5} = 16$ , by *Scholium* to *Prop. 8*.

Therefore  $\frac{7}{8} + \frac{16}{1} = \frac{128+7}{8} = \frac{135}{8} = 16\frac{7}{8}$ , the required Sum.

## EXAMPLE XIX.

Add  $\frac{3}{4}l.$  to  $\frac{5}{8}$  of a Shilling.

Thus  $\frac{5}{8}$  of a Shilling is  $=\frac{5}{8}$  of  $\frac{1}{20}$ .

Then  $\frac{5}{8}$  of  $\frac{1}{20} = \frac{5}{160}l.$  and

Therefore  $\frac{5}{160} + \frac{3}{4} = \frac{360+20}{480} = \frac{380}{480} = \frac{19}{24}l.$  the required sum.

It would have been the same if we had reduc'd  $\frac{3}{4}l.$  to the Fraction of a Shilling, by the *Scholium* to *Prop. 8*. which is found to be  $=\frac{60}{4}s.$

For  $\frac{3}{4}$  of  $\frac{1}{20} = \frac{3 \times 20}{4} = \frac{60}{4}$  and  $\frac{60}{4} + \frac{5}{8} = \frac{360+20}{24} = \frac{380}{24} = 15s. \frac{5}{6} = 15s. 10d.$

Therefore  $\frac{19}{24}l. = 15s. \frac{5}{6} = 15s. 10d.$

## CASE VIII. EXAMPLE XX.

Add  $\frac{3}{4}$  of  $\frac{5}{8}$  to  $\frac{3}{5}$  of  $\frac{7}{8}$ .

Thus  $\frac{3}{4}$  of  $\frac{5}{8} = \frac{15}{32}$ , and  $\frac{3}{5}$  of  $\frac{7}{8} = \frac{21}{40}$ .

Then  $\frac{15}{32} + \frac{21}{40} = \frac{110+84}{960} = 1\frac{3}{20}$  the required sum.

## EXAMPLE XXI.

Add  $\frac{2}{3}$  of  $\frac{5}{11}$  of  $\frac{7}{9}$  to  $\frac{2}{7}$  of  $\frac{1}{13}$ .

Thus  $\frac{2}{3}$  of  $\frac{5}{11}$  of  $\frac{7}{9} = \frac{70}{297}$ ; and  $\frac{2}{7}$  of  $\frac{1}{13} = \frac{2}{91}$ , by *Prop. 9*. Then by *Case 1*.  $\frac{70}{297} + \frac{2}{91} = \frac{224+63}{2673} = \frac{287}{2673}$ .

## CASE IX. EXAMPLE XXII.

Add 24 and  $12\frac{1}{2}$  of  $\frac{2}{3}$ .

Thus  $24 + 12 = 36$ .

And

And  $\frac{1}{2}$  of  $\frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$ .

Therefore  $36 + \frac{1}{3} = 36\frac{1}{3}$  the Sum.

EXAMPLE XXIII.

Add 19 and  $7\frac{1}{2}$  of  $\frac{2}{7}$  together.

Thus  $19 + 7 = 26$ .

And  $\frac{1}{2}$  of  $\frac{2}{7} = \frac{1 \times 2}{2 \times 7} = \frac{1}{7}$ .

Then  $26\frac{1}{7}$  is the Sum.

CASE X. EXAMPLE XXIV.

Add  $\frac{1}{3}$  of 95 and  $\frac{7}{8}$  of 14 together.

Thus  $\frac{1}{3}$  of  $95 = 31\frac{2}{3}$ .

And  $\frac{7}{8}$  of  $14 = 9\frac{7}{4}$ .

Then  $31\frac{2}{3} + 9\frac{7}{4} = \frac{105}{3} + \frac{35}{2} = \frac{210}{6} + \frac{105}{6} = \frac{315}{6} = 52\frac{3}{2} = 52\frac{1}{2}$ , the Sum.

EXAMPLE XXV.

Add  $\frac{1}{2}$  of 100 and  $\frac{3}{4}$  of 105 together.

Thus  $\frac{1}{2}$  of  $100 = 50$ .

And  $\frac{3}{4}$  of  $105 = 31\frac{3}{4}$ .

Therefore  $50 + 31\frac{3}{4} = 81\frac{3}{4} = 81\frac{3}{4}$  the Sum.

Here the Reader may observe, that if any of the Fractions to be added be compound ones, they must be reduced to simple Fractions, which is easily done by the Rules of Reduction, and then to be added.

I shall only give another Example or two, in a Method something different from what I have laid down, and then make a Transition to Subtraction.

PROPOSITION XV.

**T**O add to any Fraction any Part or Parts of the same Fraction.

R U L E.

*Add the Denominator to the Numerator of the Fraction expressing the Part or Parts to be added, and that Sum multiply by the other Numerator, and this shall be a new*

E 2.

*Numerator.*

*Numerator ; then multiply the two Denominators together, and the Product is a new Denominator.*

## EXAMPLE I.

To  $\frac{3}{5}$  add  $\frac{4}{9}$  of the same  $\frac{3}{5}$ .

Thus  $4 + 9 = 13$ , and  $13 \times 3 = 39$  Numerator.

And  $5 \times 9 = 45$  Denominator.

Therefore  $\frac{39}{45} = 1\frac{1}{5}$  the required Sum.

## EXAMPLE II.

To  $\frac{1}{17}$  add  $\frac{4}{9}$  of the same  $\frac{1}{17}$ .

Thus  $4 + 5 = 9$ , and  $9 \times 11 = 99$  Numerator.

And  $17 \times 5 = 85$  Denominator.

Therefore  $\frac{99}{85} = 1\frac{14}{85}$  the required Sum.

## EXAMPLE III.

To  $\frac{3}{4}l.$  add  $\frac{3}{4}$  of the same  $\frac{3}{4}$ .

Thus  $5 + 3 = 8$ , and  $8 \times 3 = 24$  Numerator.

And  $4 \times 5 = 20$  Denominator.

Therefore  $\frac{24}{20} = 1l. \frac{1}{5} = 1l. 4s.$

For  $15s. + 9s. = 24s. = 1l. 4s.$





# SUBTRACTION.

## PROPOSITION XVI.

**I**F the Denominators are not equal, they must be reduced to such as have equal ones, managed as in Addition ; then by this

### RULE.

*The Difference of the Numerators, set over the common Denominator, shall be the Difference of the given Fractions.*

#### CASE I. EXAMPLE I.

Let  $\frac{2}{9}$  be taken from  $\frac{5}{9}$ .

$$\text{Then } \frac{5}{9} - \frac{2}{9} = \frac{3}{9} = \frac{1}{3}.$$

#### DEMONSTRATION.

For  $\frac{5}{9} : \frac{2}{9} :: 5 : 2$ , by Cor. 3. pag. 14.

$$\text{And } \frac{5}{9} - \frac{2}{9} : \frac{2}{9} :: 5 - 2 : 2.$$

$$\text{But } \frac{5-2}{9} : \frac{2}{9} :: 5-2 : 2.$$

$$\text{Therefore } \frac{5}{9} - \frac{2}{9} : \frac{2}{9} :: \frac{5-2}{9} : \frac{2}{9}.$$

$$\text{Conseq. } \frac{5}{9} - \frac{2}{9} = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}.$$

#### CASE II. EXAMPLE II.

From  $\frac{5}{6}$  take  $\frac{1}{3}$ .

Thus  $6 \times 3 = 18$  Denominator.

$5 \times 3 = 15$  Numerator.

$6 \times 1 = 6$  Numerator.

Then  $\frac{15-6}{18} = \frac{9}{18} = \frac{1}{2}$ , as is evident from Example 1.

in Addition.

## Of VULGAR FRACTIONS.

## EXAMPLE III.

Take  $\frac{2}{7}$  from  $\frac{5}{9}$ .

Thus  $7 \times 9 = 63$  Denominator:  
 $7 \times 5 = 35$  a Numerator.  
 $3 \times 9 = 27$  a Numerator.

Then  $\frac{35-27}{63} = \frac{8}{63}$ , Answer.

## CASE III. EXAMPLE IV.

From  $34\frac{1}{3}$  take 13.Thus  $34 - 13 = 21$ .And  $\frac{1}{3} - 0 = \frac{1}{3}$ .

Therefore  $21\frac{1}{3}$  is the required Remainder, as is evident by Inspection, and *Example 5.* in Addition being a Proof thereof.

## EXAMPLE V.

From  $29\frac{5}{11}$  take 18.

Thus  $29\frac{5}{11} - 18 = 11\frac{5}{11}$  the required Remainder.

## CASE IV. EXAMPLE VI.

From 47 take  $3\frac{3}{11}$ .Thus  $11 - 3 = \frac{8}{11}$  Numerator.And  $47 - 1 = 46$ .

Then  $46\frac{8}{11}$  is the required Remainder.

*Note,* When you would Subtract a Vulgar Fraction from an Integer or whole Number; first subtract the Numerator from its Denominator, and the Remainder is a new Numerator to its Denominator; then carry 1 to be taken from the whole Number, and the Remainder is the required Fraction. An Example or two will render this very familiar.

## EXAMPLE VII.

From 67 take  $13\frac{1}{9}$ .Thus  $19 - 13 = \frac{6}{9}$  Numerator.And  $67 - 1 = 66$ .

Therefore  $66\frac{6}{9}$  is the required Remainder.



EXAMPLE VIII.

From  $429$  take  $3\frac{1}{9}$ .

First  $49 - 31 = 18$ .

And  $429 - 1 = 428$ .

Therefore  $428\frac{18}{9}$  is the required Remainder.

EXAMPLE IX.

From  $41\frac{1}{10}$  take  $34\frac{1}{3}$ .

Thus  $41 - 34 = 7$ .

And  $\frac{1}{10} - \frac{1}{3} = \frac{30 - 10}{30} = \frac{20}{30} = \frac{2}{3}$ .

Therefore  $7 - \frac{2}{3} = 6\frac{1}{3}$ , the required Fraction, as is evident from *Ex. 9.* in Addition.

EXAMPLE X.

From  $14\frac{2}{3}$  take  $12\frac{2}{3}$ .

Thus  $14 - 12 = 2$ .

And  $\frac{2}{3} - \frac{2}{3} = \frac{12 - 10}{15} = \frac{2}{15}$ .

Therefore  $2\frac{2}{15}$  is the required Remainder.

EXAMPLE XI.

From  $81\frac{5}{9}$  take  $49\frac{2}{3}$ .

Thus  $81 - 49 = 32$ .

And  $\frac{5}{9} - \frac{2}{3} = \frac{35 - 27}{63} = \frac{8}{63}$ .

Therefore  $32 + \frac{8}{63} = 32\frac{8}{63}$ , the required Remainder.

CASE VI. EXAMPLE XII.

From  $13\frac{1}{3}$  take  $\frac{1}{3}$ .

Thus  $\frac{1}{3} - \frac{1}{3} = \frac{9 - 8}{24} = \frac{1}{24}$ .

Then  $13 - \frac{1}{24} = 12\frac{23}{24}$  is the required Remainder, as is manifest from the *Note* in *Ex. 6.* above.

## CASE VII. EXAMPLE XIII.

From  $\frac{1}{4}$  take  $\frac{2}{3}$  of  $\frac{8}{9}$ .Thus  $\frac{2}{3}$  of  $\frac{8}{9} = \frac{16}{27}$ .

Then  $\frac{1}{4} - \frac{16}{27} = \frac{351 - 224}{378} = \frac{127}{378}$  the required Remainder.

## CASE VIII. EXAMPLE XIV.

Take  $\frac{3}{4}$  of  $\frac{5}{8}$  from  $\frac{2}{3}$  of  $\frac{1}{2}$ .First  $\frac{3}{4}$  of  $\frac{5}{8} = \frac{15}{32}$ .And  $\frac{2}{3}$  of  $\frac{1}{2} = \frac{1}{3}$ .

Then  $\frac{1}{3} - \frac{15}{32} = \frac{225 - 216}{540} = \frac{9}{540} = \frac{1}{60} = \frac{1}{60}$  the required Remainder.

## EXAMPLE XV.

From  $\frac{1}{2}$  of  $\frac{6}{7}$  take  $\frac{7}{8}$  of  $\frac{1}{11}$ .First  $\frac{1}{2}$  of  $\frac{6}{7} = \frac{3}{7}$ .And  $\frac{7}{8}$  of  $\frac{1}{11} = \frac{7}{88}$ .

Then  $\frac{3}{7} - \frac{7}{88} = \frac{2112 - 980}{3080} = \frac{1132}{3080} = \frac{284}{770}$  the required Remainder.

## CASE IX. EXAMPLE XVI.

From  $\frac{3}{4}$  and  $\frac{2}{3}$  take  $\frac{7}{8}$  and  $\frac{9}{10}$ .First  $\frac{3}{4}$  and  $\frac{2}{3} = \frac{9+8}{12} = \frac{17}{12}$ .And  $\frac{7}{8}$  and  $\frac{9}{10} = \frac{72+70}{80} = \frac{142}{80}$ .

Then  $\frac{17}{12} - \frac{142}{80} = \frac{1704 - 1360}{960} = \frac{344}{960} = \frac{43}{120}$ , the required Remainder.

## EXAMPLE XVII.

From  $\frac{1}{3}$  and  $\frac{2}{7}$  take  $\frac{3}{5}$  and  $\frac{4}{11}$ .

Thus

$$\text{Thus } \frac{1}{3} \text{ and } \frac{2}{7} = \frac{9+7}{21} = \frac{16}{21}.$$

$$\text{And } \frac{3}{5} \text{ and } \frac{4}{11} = \frac{33+20}{55} = \frac{53}{55}.$$

$$\text{Then } \frac{16}{21} - \frac{53}{55} = \frac{1113-880}{1155} = \frac{233}{1155} \text{ the required Remainder.}$$

If these Examples be well understood, the whole Business of *adding* and *subtracting Vulgar Fractions* will be easy; which I think really much more difficult to perform than either Multiplication or Division, as will appear by the next Propositions.





# MULTIPLICATION.

## PROPOSITION XVII.

**T**O multiply one Fraction by another, multiply the Numerators into one another, and likewise the Denominators, thus  $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$ .

## DEMONSTRATION.

• This is also Division; for to multiply  $\frac{2}{3}$  by  $\frac{1}{2}$ , is the same as to take the half of  $\frac{2}{3}$  Parts; and therefore if  $\frac{4}{3}$  be multiplied by  $\frac{2}{3}$ , we must find 2 third Parts of 4 Fifths; then  $\frac{4}{3}$  being the Dividend, it must be divided into as many equal Parts as the Denominator of the Multiplier hath Unities, viz. 3, and that Part must be multiplied by the Numerator 2. Again, if  $\frac{3}{4}$  be multiplied by  $\frac{2}{3}$  it will be  $\frac{4 \times 2}{4 \times 3} = \frac{8}{12} = \frac{2}{3}$  by the 2d following *Lemma*.

And  $\frac{6}{12} : \frac{8}{12} :: 6 : 8$  by *Corollary 3. pag. 14.*

That is  $\frac{6}{12} : \frac{2}{3} :: 3 : 4$ .

Or,  $\frac{6}{12} : \frac{2}{3} :: \frac{3}{4} : 1$ .

Therefore  $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$  by nature of Proportion.

## SCHOLIUM I.

Seeing this is Division upon the Matter, it appears why the Product becomes less, therefore the Product of two proper Fractions must be less than either of them. Thus  $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$ , and  $\frac{10}{21}$  is less than either  $\frac{2}{3}$  or  $\frac{5}{7}$ .

For  $1 : \frac{7}{7} :: \frac{2}{3} : \frac{10}{21}$ , by the Nature of Multiplication.

## SCHOLIUM II.

The Product of any Quantity, multiplied by a proper Fraction, is always less than that Quantity.

For in multiplying by an Unit, the Product will be equal to the Multiplicand.

But

But a less Multiplier gives a less Product.

Therefore multiplying by a proper Fraction (*i. e.* by less than Unit) the Product must be less than the Multiplicand.

*Note,* If mixt Fractions are to be multiplied together, reduce them to improper ones.

If compound Fractions are to be multiplied together, reduce them to single ones.

If one of the Factors be a whole Number, it must be made an improper Fraction.

But before I give Examples in multiplying Fractions one by another, it will be necessary, in order to give the Learner a clear Idea of the Reason and Certainty of such Rules, to premise the two following *Lemmas*.

### L E M M A I.

To multiply the Numerator, is to multiply the Fraction; thus  $\frac{2 \times 3}{5} = \frac{2}{5} \times 3$ .

$$\text{For } 5 : 2 :: 3 : \frac{2 \times 3}{5}$$

$$\text{And } 5 : 2 :: 1 : \frac{2}{5}$$

$$\text{Therefore } 1 : \frac{2}{5} :: 3 : \frac{2 \times 3}{5}$$

$$\text{But } 1 : \frac{2}{5} :: 3 : \frac{2}{5} \times 3$$

$$\text{Therefore } \frac{2 \times 3}{5} = \frac{2}{5} \times 3$$

### L E M M A II.

The Terms of a Fraction being multiplied or divided by the same Quantity alter not its Value.

$$\text{Thus } \frac{2 \times 3}{2 \times 4} = \frac{6}{8}$$

$$\text{For } 2 \times 3 (=6) : 2 \times 4 (=8) :: 3 : 4$$

$$\text{Therefore } \frac{3}{4} = \frac{6}{8}$$

$$\text{Also } \frac{5 \overline{)10}(2}{5 \overline{)15}(3} = \frac{2}{3}$$

$$\text{For } \frac{10}{5} : \frac{15}{5} :: 10 : 15, \text{ by Corollary 3. pag. 14.}$$

$$\text{But } \frac{10}{5} : \frac{15}{5} :: 2 : 3$$

$$\text{Therefore } \frac{10}{15} = \frac{2}{3}$$

• Seeing

Seeing (I hope) I have fully explained the Nature of Multiplication, and demonstrated its Rules sufficiently, it will be proper now to give some Examples.

## CASE I. EXAMPLE I.

$$\frac{3}{5} \times \frac{2}{3}$$

Thus  $\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$  the required Product.

## EXAMPLE II.

Multiply  $\frac{11}{17}$  by  $\frac{18}{37}$ .

$$\text{Thus } 11 \times 18 = 198.$$

$$\text{And } 17 \times 37 = 629.$$

Therefore  $\frac{11 \times 18}{17 \times 37} = \frac{198}{629}$  the required Product.

## CASE II. EXAMPLE III.

Multiply 12 by  $3\frac{3}{5}$ .

$$\text{Thus } 3\frac{3}{5} = \frac{3 \times 5 + 3}{5} = \frac{18}{5}, \text{ by Prop. 5.}$$

Then  $\frac{12 \times 18}{1 \times 5} = \frac{216}{5} = 43\frac{1}{5}$  the required Product.

## EXAMPLE IV.

Multiply 171 by  $56\frac{35}{71}$ .

$$\text{Thus } 56\frac{35}{71} = \frac{56 \times 71 + 35}{71} = \frac{4011}{71}.$$

Then  $\frac{171 \times 4011}{1 \times 71} = \frac{685881}{71} = 9660\frac{21}{71}$  the required Product.

## CASE III. EXAMPLE V.

Multiply 11 by  $\frac{5}{7}$ .

Thus  $\frac{11 \times 5}{1 \times 7} = \frac{55}{7} = 7\frac{6}{7}$  the required Product.

Ex -

EXAMPLE VI.

Multiply  $15\frac{1}{2}$  by  $12\frac{1}{2}$ .

Thus  $\frac{15 \times 12}{1 \times 2} = \frac{180}{2} = 90$  the required Product.

CASE IV. EXAMPLE VII.

Multiply  $17\frac{5}{9}$  by  $47$ .

$$\text{Thus } 17\frac{5}{9} = \frac{17 \times 9 + 5}{9} = \frac{158}{9}.$$

And  $\frac{158 \times 47}{9 \times 1} = \frac{7426}{9} = 825\frac{1}{9}$  the required Product.

EXAMPLE VIII.

Multiply  $51\frac{2}{3}$  by  $13$ .

$$\text{Thus } 51\frac{2}{3} = \frac{51 \times 3 + 2}{3} = \frac{155}{3}.$$

And  $\frac{155 \times 13}{3 \times 1} = \frac{2015}{3} = 671\frac{2}{3}$ , the required Product.

CASE V. EXAMPLE IX.

Multiply  $11\frac{1}{3}$  by  $3\frac{2}{3}$ .

$$\text{First } 11\frac{1}{3} = \frac{11 \times 3 + 1}{3} = \frac{34}{3}.$$

$$\text{And } 3\frac{2}{3} = \frac{3 \times 3 + 2}{3} = \frac{11}{3}.$$

Then  $\frac{34 \times 11}{3 \times 3} = \frac{374}{9} = 41\frac{5}{9}$ , the required Product.

EXAMPLE X.

Multiply  $17\frac{1}{2}$  by  $17\frac{1}{2}$ .

Thus  $17\frac{1}{2} = \frac{17 \times 2 + 1}{2} = \frac{35}{2}.$

$$\text{Then } 17\frac{1}{2} = \frac{17 \times 2 + 1}{2} = \frac{35}{2}.$$

Therefore  $\frac{71 \times 71}{4 \times 4} = \frac{5041}{16} = 315 \frac{1}{16}$  the required Product.

## EXAMPLE XI.

Multiply  $584 \frac{117}{324}$  by  $81 \frac{17}{17}$ .

$$\text{First } 584 \frac{117}{324} = \frac{584 \times 524 + 117}{524} = \frac{306133}{524}.$$

$$\text{And } 81 \frac{17}{17} = \frac{81 \times 17 + 11}{17} = \frac{1388}{17}.$$

Therefore  $\frac{306133}{524} \times \frac{1388}{17} = \frac{424912604}{8908} = 47700 \frac{1004}{8908}$   
the required Product.

## CASE VI. EXAMPLE XII.

Multiply  $17 \frac{7}{7}$  by  $\frac{5}{9}$ .

$$\text{Thus } 17 \frac{7}{7} = \frac{17 \times 7 + 3}{7} = \frac{122}{7}.$$

And  $\frac{122}{7} \times \frac{5}{9} = \frac{610}{63} = 9 \frac{44}{63}$ , the required Product.

## EXAMPLE XIII.

Multiply  $5 \frac{5}{5}$  by  $\frac{7}{7}$ .

$$\text{Thus } 5 \frac{5}{5} = \frac{5 \times 5 + 3}{5} = \frac{28}{5}.$$

And  $\frac{28}{5} \times \frac{7}{7} = \frac{196}{35} = 5 \frac{12}{35}$  the required Product.

## CASE VII. EXAMPLE XIV.

Multiply  $\frac{21}{33}$  by  $37$ .

Thus  $\frac{21}{33} \times 37 = \frac{777}{33} = 23 \frac{1}{3}$ , the required Product.

## EXAMPLE XV.

Multiply  $\frac{7}{7}$  by  $39$ .

Thus  $\frac{7}{7} \times 39 = \frac{273}{7} = 39$ , the required Product.

This being the same as Case 3. I shall not insist any more on it, seeing such Operations are evidently easy by Inspection, and therefore shall proceed to



CASE VIII. EXAMPLE XVI.

Multiply  $\frac{3}{7}$  by  $11\frac{1}{3}$ .

$$\text{Thus } 11\frac{1}{3} = \frac{11 \times 5 + 3}{5} = \frac{58}{5}.$$

$$\text{And } \frac{58}{5} \times \frac{3}{7} = \frac{174}{35} = 4\frac{14}{35}, \text{ the required Product.}$$

EXAMPLE XVII.

Multiply  $1\frac{1}{3}$  by  $17\frac{1}{3}$ .

$$\text{Thus } 17\frac{1}{3} = \frac{17 \times 27 + 13}{27} = \frac{472}{27}.$$

$$\text{And } \frac{472}{27} \times \frac{4}{13} = \frac{1888}{351} = 14\frac{278}{351}, \text{ the required Product.}$$

This being the same as Case 5. shall not trouble the Reader with any more in this Case.

CASE IX. EXAMPLE XVIII.

Multiply  $\frac{2}{3}$  of  $\frac{3}{4}$  by  $\frac{5}{6}$  of  $\frac{6}{7}$ .

$$\text{Thus } \frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12} = \frac{1}{2}.$$

$$\text{And } \frac{5}{6} \text{ of } \frac{6}{7} = \frac{30}{42} = \frac{5}{7}.$$

$$\text{Then } \frac{6}{12} \times \frac{30}{42} = \frac{180}{504} = \frac{5}{14}.$$

$$\text{So } \frac{1}{2} \times \frac{5}{7} = \frac{5}{14}, \text{ the required Product.}$$

CASE X. EXAMPLE XIX.

Multiply  $\frac{3}{4}$  and  $\frac{5}{6}$  by  $\frac{2}{3}$  and  $\frac{6}{11}$ .

$$\text{Thus } \frac{3}{4} \text{ and } \frac{5}{6} = \frac{25 + 27}{45} = \frac{52}{45}.$$

$$\text{And } \frac{2}{3} \text{ and } \frac{6}{11} = \frac{18 + 22}{33} = \frac{40}{33}.$$

$$\text{Then } \frac{52}{45} \times \frac{40}{33} = \frac{2080}{1485} = 1\frac{112}{297}, \text{ the required Product.}$$

A QUESTION.

What Number is  $\frac{2}{7}$  of 21.

$$\text{Thus } \frac{2}{7} \times 21 = \frac{42}{7} = 6.$$

$$\text{For } 1 : \frac{2}{7} :: 21 : 6.$$

$$\text{Or } 7 : 2 :: 21 : 6.$$

Here, you see, is no Difficulty in managing Vulgar Fractions in Multiplication, if so that you observe the Method of reducing them as was done in Addition and Subtraction.

# D I V I S I O N.

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## P R O P O S I T I O N XVIII.

**T**O divide a Fraction by another, as  $\frac{4}{3}$  by  $\frac{2}{3}$ .  
 Invert the Denominator, that is, make it  $\frac{3}{2}$ , then multiply the Dividend by it, and it's  $\frac{12}{6}$  which is the Divisor.

## D E M O N S T R A T I O N.

Because the Divisor is to the Dividend, as Unity to the Quotient, then the Dividend will be to the Divisor, as the Quotient to Unity; and if the Fractions be brought to the same Denominator, seeing they are equal to the Quotients from the Division of the Numerator by the Denominator, the Numerator of the divided Fraction will be to that of the dividing, as the Fraction divided to the one dividing, and so the Numerator of the Dividend is to that of the Divisor, as the Quotient to Unity; wherefore the Fractions being brought to one Denominator, the Numerator of the Dividend is to be divided by the Numerator of the Divisor to obtain the Quotient, and it will be  $\frac{12}{6}$ . Therefore for the Division of Common Fractions, this is the

## R U L E.

*Multiply the Dividend by the Divisor's reverse Fraction, or which is the same, imagine the Terms of the Divisor changed, then work as in Multiplication.*

## E X A M P L E.

Thus  $\frac{2}{3} \frac{4}{3} (= \frac{12}{6} = \frac{6}{3} = 1 \frac{1}{3})$ .

That is  $\frac{2}{3} \times \frac{4}{3} = \frac{12}{6}$ .

For  $1 : \frac{2}{3} :: \frac{4}{3} : \frac{12}{6}$ .

But  $\frac{2}{3} : 1 :: 1 : \frac{3}{2}$  by Cor. 4. pag. 14.

And  $\frac{2}{3} : 1 :: \frac{4}{3} : \frac{12}{6}$ .

There-

Therefore  $\frac{4}{3} \div \frac{2}{3} = 1\frac{2}{3}$ , by the Nature of Proportion.

If either *Dividend* or *Divisor* be *whole* or *mixt* Numbers, or if both be mixt Numbers, *reduce them to improper Fractions.*

If they are Compound Fractions, *reduce them to single ones.*

If the *Fractions* are of one *Denomination*,

Then cast off that Denominator, and divide the *Numerators.*

Because *Fractions* having the same *Denominators*, are as their Numerators. Cor. 3. pag. 14.

# S C H O L I U M.

The *Quotient* of any *Quantity* divided by a *proper Fraction* is always greater than that *Quantity.*

For in *dividing* by *Unit*, the *Quotient* will be equal to the *Dividend.*

But a *less Divisor* gives a *greater Quotient.*

Therefore in *dividing* by a *proper Fraction*, (that is, by less than an Unit,) the *Quotient* must be greater than the *Dividend.*

Or thus  $\frac{2}{3} \div \frac{2}{3} (= 1\frac{2}{3} = \frac{5}{3}$ , and  $\frac{5}{3}$  is greater than  $\frac{2}{3}$ .

For  $1 : \frac{2}{3} :: \frac{2}{3} : \frac{4}{9}$ , by the Nature of Division.

And  $1 : \frac{2}{3} :: \frac{3}{2} : \frac{4}{3}$  by *Alternation.*

But  $1 > \frac{2}{3}$ , therefore  $\frac{3}{2} > \frac{4}{3}$ .

Note,  $>$  and  $<$  are signs signifying greater and lesser.

## CASE I. EXAMPLE I.

Divide  $1\frac{1}{2}$  by  $\frac{2}{3}$ .

First invert the Divisor  $\frac{2}{3}$  and it will be  $\frac{3}{2}$ .

Then  $\frac{1}{2} \div \frac{3}{2} (= \frac{1}{2} \times \frac{2}{3} = \frac{1}{3})$  the *Quotient* required.

## EXAMPLE II.

Divide  $\frac{2}{3}$  by  $\frac{4}{5}$ .

Thus  $\frac{2}{3} \div \frac{4}{5} (= \frac{2}{3} \times \frac{5}{4} = 1\frac{1}{6})$ , the *Quotient* required.

## CASE II. EXAMPLE III.

Divide  $43\frac{1}{3}$  by 12.Thus  $43\frac{1}{3} = \frac{43 \times 3 + 1}{3} = \frac{216}{3}$ ; then  $\frac{1}{12}$  inverted,And  $\frac{1}{12} \times \frac{216}{3} (= \frac{216}{36} = 6)$ , the required Quotient, as is evident from Case 2. Ex. 3. in Multiplication.

## EXAMPLE IV.

Divide  $9660\frac{2}{71}$  by 171.Thus  $9660\frac{2}{71} = \frac{9660 \times 71 + 2}{71} = \frac{685882}{71}$ .And 171 order'd  $= \frac{171}{1}$ .Then  $\frac{1}{171} \times \frac{685882}{71} (= 57\frac{35}{71})$ , the required Quotient, as appears from Multiplication.

## CASE III. EXAMPLE V.

Divide  $7\frac{6}{7}$  by  $\frac{5}{7}$ .Thus  $7\frac{6}{7} = \frac{7 \times 7 + 6}{7} = \frac{55}{7}$ .And  $\frac{7}{5} \times \frac{55}{7} (= \frac{385}{35} = 11)$ , the Quotient required. See Multiplication, Case 3.

## EXAMPLE VI.

Divide  $139\frac{93}{121}$  by  $\frac{112}{121}$ .Thus  $139\frac{93}{121} = \frac{139 \times 121 + 93}{121} = \frac{16819 + 93}{121} = \frac{16912}{121}$ .And  $\frac{121}{112} \times \frac{16912}{121} (= 151)$ , the Quotient required.

## CASE IV. EXAMPLE VII.

Divide  $825\frac{1}{9}$  by 47.Thus  $825\frac{1}{9} = \frac{825 \times 9 + 1}{9} = \frac{7426}{9}$ .Then  $\frac{1}{47} \times \frac{7426}{9} (= \frac{7426}{423} = 17\frac{5}{9})$ , the required Quotient.

EXAMPLE VIII.

Divide  $16\frac{3}{11}$  by 4.

$$\text{Thus } 16\frac{3}{11} = \frac{16 \times 11 + 3}{11} = \frac{176 + 3}{11} = \frac{179}{11}.$$

Then  $\frac{1}{4} \cdot \frac{179}{11} (= \frac{179}{44} = 4\frac{3}{44}$ , the required Quotient.

CASE V. EXAMPLE IX.

Divide  $42\frac{2}{21}$  by  $3\frac{5}{7}$ .

$$\text{Thus } 42\frac{2}{21} = \frac{42 \times 21 + 2}{21} = \frac{882 + 2}{21} = \frac{884}{21}.$$

$$\text{And } 3\frac{5}{7} = \frac{3 \times 7 + 5}{7} = \frac{21 + 5}{7} = \frac{26}{7}.$$

Then  $\frac{7}{26} \cdot \frac{884}{21} (= \frac{6188}{546} = 11\frac{2}{3}$ , the required Quotient.

EXAMPLE X.

Divide  $19\frac{13}{17}$  by  $7\frac{23}{31}$ .

$$\text{Thus } 19\frac{13}{17} = \frac{19 \times 17 + 13}{17} = \frac{323 + 13}{17} = \frac{336}{17}.$$

$$\text{And } 7\frac{23}{31} = \frac{7 \times 31 + 23}{31} = \frac{217 + 23}{31} = \frac{240}{31}.$$

Then  $\frac{31}{240} \cdot \frac{336}{17} (= \frac{10416}{4080} = 2\frac{47}{85}$ , the required Quotient.

CASE VI. EXAMPLE XI.

Divide  $17\frac{3}{5}$  by  $\frac{5}{9}$ .

$$\text{Thus } 17\frac{3}{5} = \frac{17 \times 5 + 3}{5} = \frac{85 + 3}{5} = \frac{88}{5}.$$

Then  $\frac{9}{5} \cdot \frac{88}{5} (= \frac{792}{25} = 31\frac{17}{25}$ , the Quotient required.

EXAMPLE XII.

Divide  $29\frac{11}{13}$  by  $\frac{13}{19}$ .

$$\text{Thus } 29\frac{11}{13} = \frac{29 \times 13 + 11}{13} = \frac{377 + 11}{13} = \frac{188}{13}.$$

Then  $\frac{19}{13} \cdot \frac{188}{13} (= \frac{11252}{169} = 66\frac{28}{169}$ , the required Quotient.

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## CASE VII. EXAMPLE XIII.

Divide  $\frac{21}{11}$  by 37.Thus  $\frac{1}{11} \times \frac{21}{11} (= \frac{21}{121})$ , the required Quotient.

## EXAMPLE XIV.

Divide  $\frac{1}{11}$  by 19.Thus  $\frac{1}{11} \times \frac{1}{11} (= \frac{1}{121})$ , the required Quotient.

## CASE VIII. EXAMPLE XV.

Divide  $\frac{1}{2}$  by  $17\frac{1}{2}$ .Thus  $17\frac{1}{2} = \frac{17 \times 2 + 1}{2} = \frac{35}{2}$ .Then  $\frac{1}{35} \times \frac{1}{2} (= \frac{1}{70})$ , the required Quotient.

## CASE IX. EXAMPLE XVI.

Divide  $\frac{1}{2}$  of  $\frac{1}{2}$  by  $\frac{1}{2}$ .Thus  $\frac{1}{2}$  of  $\frac{1}{2} = \frac{1}{4} = \frac{1}{4}$ .And  $\frac{1}{4} \times \frac{1}{2} (= \frac{1}{8})$ , the required Quotient.

## EXAMPLE XVII.

Divide  $\frac{1}{2}$  of  $\frac{1}{11}$  and  $\frac{1}{2}$  by  $\frac{1}{2}$ .Thus  $\frac{1}{2}$  of  $\frac{1}{11} = \frac{1}{22}$ .And  $\frac{1}{22} + \frac{1}{2} = \frac{60 + 231}{385} = \frac{291}{385}$ .Then  $\frac{1}{2} \times \frac{291}{385} (= \frac{291}{770} = 1\frac{51}{770})$ , the Quotient, and so any compound Fraction.

## A QUESTION.

What Number does 6 contain  $\frac{2}{7}$  of?Thus  $\frac{2}{7}$  inverted is  $\frac{7}{2}$  and  $\frac{7}{2} \times 6 (= 21)$ .For  $\frac{2}{7} : 1 :: 6 : 21$ .Or  $2 : 7 :: 6 : 21$ , Answer.

By what has been done, may be perform'd all manner of *Additions, Subtractions, Multiplications* and *Divisions*

*ions* in Fractions. The Reason why I have taken the Numbers so small in the foregoing Examples, was on purpose, that I might not perplex the Mind of the Learner, since little familiar Examples are sufficient, and more proper to enlighten the Understanding than larger ones.

Methinks it would not be improper, if we come now to put the preceding Rules into Use, and explain a little the Nature of Fractions in answering Questions.

But before I begin with Questions relating to Proportion, as in the Rule of Three Direct, &c. it will be requisite to premise the following Proposition, which will be found useful in Questions of other Kinds.

### P R O P O S I T I O N XIX.

**W**HEN Number is applied to any Quantity, then that Quantity doth, or is imagined, to contain so many equal Quantities of the same Kind, as the Number applied doth contain Units; so that of Quantities of one Kind compared together, when the one is said to be one Number, the other another Number, the expressing and naming of the *Ratio* which is between them, is nothing but to find a Number, which shall have such *Ratio* to 1, as of the two Numbers applied the greater hath to the lesser, which in consideration that the Comparison required is to 1, if of the Numbers applied you divide the greater by the lesser, the Quotient will be the Number you desire, expressing the *Ratio* demanded; and is named by saying the Number found, adding this Word *times*, if the Quantities compared be in the greater Inequality; but if in the lesser with this Word *under* put before, to make Distinction of the one and the other.

### E X A M P L E.

3 to 2 is  $1\frac{1}{2}$  times.

For  $\frac{3}{2} = 1\frac{1}{2}$ .

Also 5 to 3 is  $1\frac{2}{3}$  times.

For  $\frac{5}{3} = 1\frac{2}{3}$ .

And

And 15 to 4 is  $3\frac{3}{4}$  times.

For  $\frac{15}{4} = 3\frac{3}{4}$ .

But 2 to 3 is *under*  $1\frac{1}{2}$  times.

And 3 to 5 is *under*  $1\frac{2}{3}$  times.

Also 3 to 7 is *under*  $2\frac{1}{3}$  times.

And 4 to 15 is *under*  $3\frac{3}{4}$  times.

Moreover 2 to 1 is 2 times.

6 to 2 is 3 times.

12 to 3 is 4 times.

But 1 to 2 is *under* 2 times.

2 to 6 is *under* 3 times.

3 to 12 is *under* 4 times, &c. in infinitum.

Ever in all Things effectually respecting what is intended to be expressed, or signified by the written Figures, Points, Pricks, Lines, &c. As in this place the intent is to shew, that of two Quantities, or Numbers of one Kind compared in the greater Inequality, the Antecedent doth contain the Consequent, as the *Ratio* doth express; and in the lesser Inequality, that the Antecedent is contained in the Consequent, as the *Ratio* doth signify. Likewise again, when 4 Quantities given of one Kind are proportional, as the *Ratio* between the first and the second, is one and the same that is between the third and the fourth; so the *Ratio* between the first and the third, of the same Quantities so given is one and the same that is between the second and the fourth. Further, that that *Ratio* which is between the Quantities of one Kind compared together, the same *Ratio* is between the Part or Parts of the same Antecedent, and the like Part or Parts of the same Consequent.





# PROPOSITION XX.

## The RULE OF THREE Direct.

### PROBLEM.

**THREE** Numbers being given to find a fourth.  
The Rule is the same as in whole Numbers.

#### EXAMPLE I.

If the  $\frac{1}{2}$  of a Yard cost  $\frac{1}{2}l.$  what will  $\frac{2}{3}$  Yard cost.

Thus, if  $\frac{1}{2} : \frac{1}{2} :: \frac{2}{3} : \frac{2}{3}$ .

For  $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ ,

And  $\frac{1}{3} \div \frac{1}{2} (\frac{2}{3}) = \frac{2}{3} = 8s. 10d. \frac{2}{3}$ .

#### EXAMPLE II.

If  $\frac{2}{3}l.$  buy  $\frac{1}{2}$  Yard, what will  $\frac{1}{2}l.$  buy?

Thus  $\frac{2}{3} : \frac{1}{2} : \frac{1}{2} : 1 \frac{1}{2}$ .

For  $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ .

And  $\frac{1}{3} \div \frac{1}{2} (\frac{2}{3}) = 1 \frac{1}{2}$  Yards.

#### EXAMPLE III.

If  $\frac{1}{2} : \frac{1}{2} : 11 : 33 \frac{1}{3}$ .

For  $\frac{1}{2} \times 11 = 7 \frac{1}{2}$ .

And  $\frac{1}{2} \div 7 \frac{1}{2} (\frac{8}{15}) = 33 \frac{1}{3}$ .

#### EXAMPLE IV.

If  $1 \frac{1}{2}$  of Hoghead of Wine cost  $31l. \frac{1}{2}$ , what will  $\frac{1}{2}$  Hoghead cost?

If  $1 \frac{1}{2} : 31 \frac{1}{2} :: \frac{1}{2} : 32 \frac{1}{2}$ .

For  $31 \frac{1}{2} = \frac{31 \times 7 + 3}{7} = \frac{217 + 3}{7} = \frac{220}{7}$ .

And  $\frac{220}{7} \times \frac{1}{2} = \frac{1100}{7}$ .

Then  $\frac{1100}{7} \div \frac{1}{2} (\frac{2}{1}) = \frac{1100}{4} = 32 \frac{1}{2} = 32l. 8s. 6d. \frac{1}{4}$ .

Ex -

## EXAMPLE V.

If  $\frac{2}{3}$  of  $\frac{1}{4}$  require 5, what will  $\frac{4}{5}$  of  $\frac{1}{12}$  require.

$$\text{If } \frac{2}{3} \text{ of } \frac{1}{4} : 5 :: \frac{4}{5} \text{ of } \frac{1}{12} : 7 \frac{1}{3}.$$

$$\text{Thus } \frac{2}{3} \text{ of } \frac{1}{4} = \frac{1}{6} = \frac{1}{2}.$$

$$\text{And } \frac{4}{5} \text{ of } \frac{1}{12} = \frac{1}{15} = \frac{1}{3}.$$

$$\text{Then } \frac{1}{15} \times \frac{5}{1} = \frac{1}{3}.$$

$$\text{And } \frac{2}{1} \times \frac{1}{3} (\frac{1}{15}) = 7 \frac{1}{3}, \text{ Answer.}$$

## EXAMPLE VI.

If  $\frac{1}{3}$  of  $\frac{2}{3}$  and  $\frac{5}{9}$  require  $\frac{8}{11}$  of  $\frac{7}{16}$  of  $\frac{5}{9}$  and  $\frac{1}{4}$ , what will 25 require?

$$\text{If } \frac{1}{3} \text{ of } \frac{2}{3} \text{ and } \frac{5}{9} : \frac{8}{11} \text{ of } \frac{7}{16} \text{ of } \frac{5}{9} \text{ and } \frac{1}{4} :: 25 : 29 \frac{21626}{34036}.$$

$$\text{First } \frac{1}{3} \text{ of } \frac{2}{3} = \frac{2}{9} = \frac{2}{3}.$$

$$\text{And } \frac{2}{3} + \frac{5}{9} = \frac{25 + 18}{45} = \frac{43}{45}.$$

$$\text{Then also } \frac{8}{11} \text{ of } \frac{7}{16} \text{ of } \frac{5}{9} = \frac{280}{1584} = \frac{35}{198}.$$

$$\text{And } \frac{35}{198} + \frac{1}{4} = \frac{594 + 140}{792} = \frac{734}{792}.$$

Now the Terms being order'd according to the preceding Rules when stated will stand thus:

$$\frac{43}{45} : \frac{734}{792} :: \frac{25}{1} : 29 \frac{21626}{34036}.$$

$$\text{For } \frac{734}{792} \times \frac{25}{1} = \frac{18350}{792}.$$

$$\text{And } \frac{18350}{792} (\frac{18350}{792}) = \frac{1009250}{34036} = 29 \frac{21626}{34036}.$$

## EXAMPLE VII.

If  $\frac{1}{3}$  and  $\frac{1}{4}$  of  $\frac{1}{3}$  require  $23 \frac{1}{3}$ , what will  $\frac{1}{7}$  of  $\frac{8}{9}$  and  $\frac{2}{3}$  require?

$$\text{Thus } \frac{1}{3} \text{ of } \frac{1}{3} = \frac{1}{9}.$$

$$\text{And } \frac{1}{3} + \frac{2}{9} = \frac{27 + 20}{60} = \frac{47}{60}.$$

$$\text{Also } 23 \frac{1}{3} = \frac{23 \times 3 + 1}{3} = \frac{69 + 1}{3} = \frac{70}{3}.$$

$$\text{Also } \frac{1}{7} \text{ of } \frac{8}{9} = \frac{8}{63}.$$

$$\text{And } \frac{8}{63} + \frac{2}{3} = \frac{24 + 126}{189} = \frac{150}{189}.$$

Then

Then  $\frac{47}{86} : \frac{10}{3} :: \frac{150}{169} : 23\frac{17073}{16649}$ .

$$\frac{150}{169} \times \frac{10}{3} = \frac{10500}{507}$$

$$\text{And } \frac{60}{47} \left( \frac{10500}{507} \right) = \frac{630000}{26649} = 23\frac{17073}{16649}$$

There is another Method whereby the Rule of-Three Direct may be performed in Vulgar Fractions, and in my Opinion is much neater, which is thus performed by observing this Rule.

### R U L E.

*The Numerator of the first Term multiplied into the Denominator of the second and third, the Product is a new Denominator; and the Denominator of the first Term multiplied into the Numerator of the second and third, the Product is a new Numerator; which new Fraction is the Answer, it being a fourth Proportional.*

An Example or two will render this familiar.

#### EXAMPLE I.

If  $\frac{1}{8}$  Yard cost  $\frac{1}{8}l$ . what will the  $\frac{1}{2}$  of a Yard cost.

1st. 2d. 3d. 4th.

$$\text{Thus } \frac{1}{8} : \frac{1}{8} :: \frac{1}{2} : 1\frac{3}{8} = \frac{11}{8}$$

$$\begin{array}{l} \text{For } 1 \times 6 \times 3 = 18 \text{ a Denominator} \\ \text{And } 8 \times 1 \times 1 = 8 \text{ a Numerator} \end{array} \left. \vphantom{\begin{array}{l} \text{For } 1 \times 6 \times 3 = 18 \text{ a Denominator} \\ \text{And } 8 \times 1 \times 1 = 8 \text{ a Numerator} \end{array}} \right\} = \frac{8}{18} \text{ or } \frac{4}{9}$$

#### EXAMPLE II.

If  $\frac{2}{9}l$ . buy  $\frac{1}{8}$  Yard, what will  $\frac{1}{2}l$ ?

$$\text{Thus } \frac{2}{9} : \frac{1}{8} :: \frac{1}{2} : 1\frac{17}{18}$$

$$\begin{array}{l} \text{For } 2 \times 8 \times 7 = 112 \text{ Denominator} \\ \text{And } 9 \times 5 \times 4 = 180 \text{ Numerator} \end{array} \left. \vphantom{\begin{array}{l} \text{For } 2 \times 8 \times 7 = 112 \text{ Denominator} \\ \text{And } 9 \times 5 \times 4 = 180 \text{ Numerator} \end{array}} \right\} = \frac{180}{112} = 1\frac{17}{14}$$

#### EXAMPLE III.

$$\text{If } \frac{1}{2} : \frac{3}{4} :: \frac{1}{11} : \frac{31}{88}$$

$$\text{For } 5 \times 7 \times 11 = 385 \text{ Denominator.}$$

$$\text{And } 9 \times 3 \times 3 = 81 \text{ Numerator.}$$

Then  $\frac{81}{385}$  is the Answer, or 4th Proportional.

As is manifest from the 1st, 2d, and 3d. Examples of this Proposition.

*A Collection of Questions promiscuously set, exercising the preceding Rules.*

### QUESTION I.

What is the  $\frac{2}{3}$  of  $\frac{1}{2}$  a Mark?

$$\text{Thus } \frac{1}{2} \text{ a Mark} = 6 \frac{s.}{8.} \quad \text{Then } 6 \frac{s.}{8.}$$

$$\begin{array}{r} 3 \\ 8 \overline{) 100} \\ \underline{26} \quad \text{Answer.} \end{array}$$

### QUESTION II.

What is the  $\frac{5}{11}$  of  $13d. \frac{1}{2}$ ?

$$\text{Thus } 1 \frac{s.}{1 \frac{d.}{2}}$$

$$\begin{array}{r} 5 \\ 11 \overline{) 57 \frac{1}{2}} \\ \underline{66 \frac{6}{11}} \end{array}$$

### QUESTION III.

What Quantity is that from which if I take  $3\frac{2}{3}$ , the Remainder shall be  $2\frac{1}{3}$ ?

$$\text{Thus } 3\frac{2}{3} = \frac{3 \times 3 + 2}{3} = \frac{9 + 2}{3} = 1\frac{1}{3}.$$

$$\text{And } 2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{6 + 1}{3} = 2\frac{1}{3}.$$

$$\text{Then } 1\frac{1}{3} + 2\frac{1}{3} = \frac{33 + 55}{15} = \frac{88}{15} = 5\frac{13}{15} \text{ the Answer.}$$

### P R O O F.

For from  $5\frac{13}{15} = \frac{88}{15}$  take  $3\frac{2}{3} = 1\frac{1}{3}$ .

$$\text{Thus } \frac{88}{15} - 1\frac{1}{3} = \frac{264 - 165}{45} = \frac{99}{45} = 2\frac{11}{15} = 2\frac{1}{3}.$$

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QUESTION IV.

What Quantity is that, to which if I add  $3\frac{4}{7}$ , the Sum shall be  $5\frac{4}{7}$ ?

$$\text{Thus from } 5\frac{4}{7} = \frac{5 \times 35 + 4}{35} = \frac{175 + 4}{35} = \frac{179}{35}$$

$$\text{Take } 3\frac{4}{7} = \frac{3 \times 7 + 4}{7} = \frac{21 + 4}{7} = \frac{25}{7}$$

$$\text{That is } \frac{179}{35} - \frac{25}{7} = \frac{1253 - 910}{245} = \frac{343}{245} = 1\frac{1}{5} \text{ Answer.}$$

QUESTION V.

A Merchant has  $\frac{1}{8}$  and  $\frac{1}{4}$  of  $\frac{1}{4}$  of a Ship; what Part is that of the Whole?

$$\text{Thus } \frac{1}{4} \text{ of } \frac{1}{4} = \frac{1}{16}, \text{ and } \frac{1}{8} + \frac{1}{16} = \frac{16 + 24}{128} = \frac{40}{128} = \frac{5}{16}.$$

QUESTION VI.

A Merchant has  $\frac{1}{3}$  of  $\frac{1}{3}$ , and  $\frac{1}{11}$  of  $\frac{2}{3}$  of the Share of the Cargo of a Ship; what Part is that of the Whole?

$$\text{Thus } \frac{1}{3} \text{ of } \frac{1}{3} = \frac{1}{9}, \text{ and } \frac{1}{11} \text{ of } \frac{2}{3} = \frac{2}{33}.$$

$$\text{Then } \frac{1}{9} + \frac{2}{33} = \frac{400 + 33}{1320} = \frac{433}{1320}.$$

QUESTION VII.

What is the Difference betwixt the Sum of  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and an Unit?

$$\text{Thus } 2 \times 3 \times 4 = 24 \text{ Denominator.}$$

$$\text{And } 1 \times 3 \times 4 = 12 \text{ Numerator.}$$

$$1 \times 2 \times 4 = 8 \text{ a Numerator.}$$

$$1 \times 2 \times 3 = 6 \text{ a Numerator.}$$

$$\text{That is } \frac{12 + 8 + 6}{24} = \frac{26}{24} = 1\frac{1}{6}.$$

$$\text{Then } 1\frac{1}{6} - 1 = \frac{1}{6} \text{ the Answer.}$$

# 64 Of VULGAR FRACTIONS.

## QUESTION VIII.

A certain Merchant having  $\frac{3}{4}$  of a Ship, sells  $\frac{3}{4}$  of his Share for 240*l*; what is the whole Ship worth?

Thus  $\frac{3}{4}$  of  $\frac{3}{4} = \frac{9}{16}$ . Then  $9 : 20 :: 240 : 533\frac{1}{3}$ .

Answer 533*l*.  $\frac{1}{3}$ .

## QUESTION IX.

A Father dying left his Son a certain Portion, of which he spent  $\frac{1}{4}$ , of the Rest he spent  $\frac{1}{2}$ , and then he had 800*l*; what was the Portion?

Thus  $1 - \frac{1}{4} = \frac{3}{4}$ , and  $\frac{1}{2}$  of  $\frac{3}{4} = \frac{3}{8}$ ; then  $\frac{3}{4} - \frac{3}{8} = \frac{3}{8}$ .

Then  $3 : 8 :: 800 : 2133\frac{1}{3}$ .

Answer 2133*l*.  $\frac{1}{3}$ .

## QUESTION X.

A younger Brother received 410*l*. which was  $\frac{3}{4}$  of  $\frac{2}{3}$  of his elder Brother's Portion; now  $3\frac{1}{2}$  times his elder Brother's Portion was  $1\frac{1}{2}$  of his Father's Estate; what was the Father's Estate?

Thus  $\frac{3}{4}$  of  $\frac{2}{3} = \frac{6}{12} = \frac{1}{2}$ . Then as  $1 : 2 :: 410 : 820$ , that is, as the Numerator of the Fraction is to its Denominator, so is 410*l*. to the elder Brother's Portion = 820.

Then  $3\frac{1}{2} = \frac{3 \times 2 + 1}{2} = \frac{7}{2}$ , and  $\frac{820}{1} \times \frac{7}{2} = \frac{5740}{2} = 2870$   
 $= 1\frac{1}{3}$  (or  $\frac{4}{3}$ ) of the Father's Estate.

That is,  $\frac{820}{1} \times \frac{7}{2} = \frac{5740}{2} = 2870$ ; now  $1\frac{1}{3}$  times 2870 = 4305*l*. the Father's Estate.

Seeing the elder Brother's Portion = 820, and as it is likewise  $1\frac{1}{3}$  of the Father's Estate, it will be

$4 : 3 :: 3\frac{1}{2}$  Brother's Portion : Father's Estate.

## QUESTION XI.

What Part of 3*d*. is  $\frac{2}{3}$  of 2*d*?

Thus  $2 : \frac{2}{3} :: 3 : \frac{4}{3}$ .

For  $\frac{2}{3} \times \frac{3}{1} = \frac{2}{1}$ , and  $(\frac{1}{3})\frac{4}{3} (= \frac{4}{9})$  Answer.

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QUESTION XII.

What Money is that in Proportion to 20s. as 5 is to  $13\frac{1}{2}$ ?

Thus  $13\frac{1}{2} : 20 :: 5 : 7\frac{1}{2}$ .

For  $20 \times 5 = 100$ , and  $13\frac{1}{2} = \frac{13 \times 2 + 1}{2} = \frac{27}{2}$ .

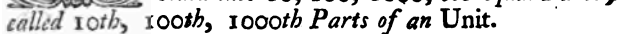
Then  $\frac{2}{27} \times 100 = 7\frac{1}{2}$  Answer,

Or 7s. 4d.  $\frac{2}{3}$ .





DEFINITION.



And as Integers increase from Unit, towards the left Hand, in a tenfold Proportion, so that a Figure in any Place is *ten* times as many in the next Place below it, and but a *tenth* Part of what it signifies in the next Place above it; therefore as the 1<sup>st</sup>, 2<sup>d</sup>, 3<sup>d</sup>, &c. Place above that of Units, is *Tens, Hundreds, Thousands, &c.* So the 1<sup>st</sup>, 2<sup>d</sup>, 3<sup>d</sup>, &c. Place below that of Units, is *Tenths, Hundredths, Thousandths, &c.* decreasing in a subdecuple Proportion; as is evident from the following Table.

Integers								Parts					
6	Millions												
5	Hundred Thousands												
4	Ten Thousands												
3	Thousands												
2	Hundreds												
1	Tens												
0	Units												
1	Tenths												
2	Hundredths												
3	Thousandths												
4	Tens of Thousandths												
5	Hundred Thousandths												
6	Millionths												

&c. Now



Now because the Denominators of Decimal Fractions differ only in the Number of Places, and not in the Figures, they being always an Unit with Cyphers, they may be expressed without their Denominators, with a Point prefixed; thus  $\frac{6}{10}$  is expressed decimally .6, and  $\frac{35}{100}$  is decimally express'd .35, also  $\frac{254}{1000}$  is express'd thus .254, where observe, this dot (.) distinguishes them from whole Numbers.

Thus  $\left\{ \begin{array}{l} .5 \\ .25 \\ .175 \end{array} \right\}$  Signifies  $\left\{ \begin{array}{l} 5 \text{ Tenths,} \\ 25 \text{ Hundredths,} \\ 175 \text{ Thousandths.} \end{array} \right.$

### C O R O L L A R Y.

As Cyphers set on the right Hand of Integers do increase the Value of them decimally, as .5, .50, .500, &c. when set on the left Hand of Decimal Fractions, they decrease the Value decimally, as .5, .05, .005, &c. But set on the left Hand of Integers, or on the right Hand of Decimal Fractions, they signify nothing, but only to fill up void Places. Thus .50000 is but 5 Tenths, and 00005. is but 5 Units.

### S C H O L I U M I.

Hence 1, 2, 3, &c. Cyphers, before a Decimal, advance it so many Places forward, whereby it is made 10, 100, 1000, &c. times less. Thus,

$\left\{ \begin{array}{l} .25 \\ .025 \\ .0025 \\ .00025 \end{array} \right\}$  Signifies  $\left\{ \begin{array}{l} 25 \text{ Hundredths,} \\ 25 \text{ Thousandths,} \\ 25 \text{ Ten Thousandths,} \\ 25 \text{ Hundred Thousandths.} \end{array} \right.$

### S C H O L I U M II.

Therefore, a Figure in the 1st, 2d, 3d, &c. decimal Place, is 10, 100, 1000, &c. times less than if it were an Integer.

### S C H O L I U M III.

Consequently, each Removal of a Figure into a Place forward

forward makes it *ten* Times less than it was before. For .05 is five hundredth Parts of an Integer, but .50 is no more than five tenth Parts of an Integer. Likewise .500, or .5000, .50000 are all the same Value, namely .5.

I have been as clear as possible in explaining the Notation of *these* Decimals, because of their vast Extensiveness, and the great Facility they bring with their Practice in several Operations, not only in Arithmetic, but in most other Parts of the Mathematics.

For arithmetical Operations may be perform'd vastly sooner by Decimal Fractions than by Vulgar Fractions, because the Denominators being omitted, the Rules of Addition, Subtraction, Multiplication and Division are performed as in whole Numbers, Regard being had to the Pointing, which is very easy: Yet; I must say, by these, Operations will not always come out exactly true; but you may come as near the Truth as possible, by bringing out more Figures.

*Regiomontanus* was the first that used Decimal Fractions in the Construction of the Tables of Sines, about A. D. 1464.

## PROPOSITION I.

**T**O reduce Vulgar Fractions into Decimals.

### R U L E.

To the Numerator add as many Cyphers as you would have Decimal Places; then divide it by the Denominator, and the Quotient (if there be no Remainder) will be a Decimal equivalent to the Vulgar Fraction given.

But when there is a Remainder, you may, by adding more Cyphers, proceed so as to bring the Quotient nearly equal.

### EXAMPLE I.

Thus  $\frac{1}{4} = \frac{1.00}{4} = .25$ , for  $4 : 1 :: 1.00 : .25$

EXAMPLE II.

$$\frac{1}{2} = \frac{1.0}{2} = .5, \text{ for } 2 : 1 :: 1.0 : .5.$$

EXAMPLE II.

$$\frac{3}{4} = \frac{3.00}{4} = .75, \text{ for } 4 : 3 :: 1.00 : .75.$$

EXAMPLE IV.

$$\frac{2}{7} = \frac{2.000000 \&c.}{7} = .285714 = \frac{2}{7} \text{ nearly, not wanting } \frac{1}{1000000} \text{ Part of an Unit. For } 7 : 2 :: 1.000000 \&c. : .285714 \&c.$$

EXAMPLE V.

$$\frac{15}{19} = \frac{15.000000 \&c.}{19} = .78947 \&c.$$

EXAMPLE VI.

$$\frac{1}{32} = \frac{1.00000}{32} = .03125.$$

*Note,* That because I annex five Cyphers to 1 the given Numerator, (as I intended to have five Places of Decimals) and there arise but four Figures in the Quotient, I must supply such Defect by prefixing as many Cyphers on the left Hand of the first Figure in the Quotient as there want Places; as in the preceding  $\frac{1}{32}$ , where the Quotient consisted but of 4 Places, here I annex a Cypher on the left Hand of 3, the first Figure in the Quotient, and then it becomes .03125 the true Decimal required.

EXAMPLE VII.

Reduce  $\frac{2}{3}$  of  $\frac{3}{4}$  into a Decimal Fraction.

$$\text{Thus } \frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12} = \frac{1}{2}, \text{ then } \frac{1.0}{2} = .5.$$

EXAMPLE VIII.

Reduce  $\frac{5}{7}$  and  $\frac{2}{3}$  of  $\frac{2}{7}$  into a Decimal Fraction.

$$\text{Thus } \frac{5}{7} \text{ of } \frac{2}{7} = \frac{6}{33} \text{ and } \frac{5}{7} + \frac{6}{33} = \frac{42+175}{245} = \frac{217}{245}.$$

$$\text{Then } \frac{217.000000 \&c.}{245} = .885714 \&c.$$

And so any Compound Fraction may be reduced into a Decimal one, by reducing it into a single one, according to the Rules of Reduction.

## COROLLARY.

These being understood, it will be no difficult Thing to find the Decimal Parts, answerable to any known Part or Parts of *Coins, Weights, Measures, &c.* if you reduce first the given Parts of *Coins, Weights, Measures, &c.* into a Vulgar Fraction, whose *Denominator* is the *Number* of those known *Parts* contained in the *Integer*, and the given *Parts* its *Numerator*. An Example or two will, I hope, render it familiar.

## EXAMPLE I.

What is the Decimal of 6s. 8d?

Thus 6s. 8d. = 80d. and 20s. = 240d. Then  $\frac{80}{240} = \frac{1}{3}$  the required Vulgar Fraction; therefore  $\frac{1.00000}{3} \&c.$  = .33333 &c.

## EXAMPLE II.

What is the Decimal of 15s?

Thus  $\frac{15}{20} = \frac{3}{4}$ , and  $\frac{3.00}{4} = .75$ .

## EXAMPLE III.

What is the Decimal of 16s. 7d  $\frac{1}{2}$ ?

Thus 16s. 7d.  $\frac{1}{2}$  = 798 Farthings, and 960 Farthings = 1l.

Then  $\frac{798}{960} = \frac{133}{160}$ ; therefore  $\frac{133.0000 \&c.}{160} = .8312 \&c.$

## EXAMPLE IV.

What is the Decimal of 19s. 11d.  $\frac{1}{4}$ ?

It is evident at Sight that 19s. 11d.  $\frac{1}{4}$  wants but *one* Farthing of a Pound (960,) and therefore it will be  $\frac{959}{960}$  the Fraction; then  $\frac{959.000000 \&c.}{960} = .998957 \&c.$

## EXAMPLE V.

What is the Decimal of  $\frac{1}{4}$ qr. 26lb. Avoirdupois Weight, when the Integer is *one Hundred Weight*?

Thus

Thus 1 C. = 112 lb. and  $\frac{3}{4}$  gr. + 26 lb. = 110 lb. then  $\frac{110}{112} = \frac{55}{56}$ ; Therefore  $\frac{55.0000 \&c.}{56} = .9821 \&c.$  the required Decimal.

EXAMPLE VI.

What is the Decimal of 5 C. 1 qr. 11 lb. the Integer being a Tun Weight?

Thus 2240 lb. = a Tun Weight, and 5 C. 1 qr. 11 lb. = 599 lb. then  $\frac{599}{2240}$  is the required Vulgar Fraction; and therefore  $\frac{599.0000 \&c.}{2240} = .2674 \&c.$  the required Decimal.

EXAMPLE VII.

What is the Decimal of 7 d.  $\frac{3}{4}$ , one Shilling being Integer?

First 7 d. is  $\frac{7}{2}$  of a Shilling, and  $\frac{3}{4}$  of a Shilling is  $\frac{3}{8}$ .

But  $\frac{7}{2} + \frac{3}{8} = \frac{336 + 36}{576} = \frac{372}{576} = \frac{31}{48}$ .

Then  $\frac{31.00000 \&c.}{48} = .64583 \&c.$  the required Decimal.

Note, If the Integer had been one foot instead of 7 d.  $\frac{3}{4}$ , it would be 7  $\frac{3}{4}$  foot, the Conclusion being the same.

EXAMPLE VIII.

What is the Decimal of 5 oz. 11 pw. 16 gr. the Integer being 1 lb?

Thus 5 oz. 11 pw. 16 gr. = 2680 gr. and 1 lb. = 5760 gr. then  $\frac{2680}{5760} = \frac{67}{144}$ .

And  $\frac{67.00000 \&c.}{144} = .46527 \&c.$  the Decimal required.

S C H O L I U M.

From what has been said may Decimal Tables be made for any Number; but I shall not in this place trouble the Reader with them, but only shew how he may make any for his Use, by a familiar Example or two.

EXAMPLE IX.

What is the Decimal of 19 s. a Pound being Integer?

Here I consider that it is  $\frac{19}{20}$ , then  $\frac{19.00}{20} = .95$ , the Decimal required. Also

Also,

<i>s.</i>					<i>s.</i>
18	is	$\frac{18}{20} = \frac{18.00}{20} = .9$	hence the Decimal of	18	$= .9$
17		$\frac{17}{20} = \frac{17.00}{20} = .85$		17	$= .85$
16		$\frac{16}{20} = \frac{16.00}{20} = .8$		16	$= .8$
15		$\frac{15}{20} = \frac{15.00}{20} = .75$		15	$= .75$
14		$\frac{14}{20} = \frac{14.00}{20} = .7$		14	$= .7$
13		$\frac{13}{20} = \frac{13.00}{20} = .65$		13	$= .65$
12		$\frac{12}{20} = \frac{12.00}{20} = .6$		12	$= .6$
11		$\frac{11}{20} = \frac{11.00}{20} = .55$		11	$= .55$
10		$\frac{10}{20} = \frac{10.00}{20} = .5$		10	$= .5$
9		$\frac{9}{20} = \frac{9.00}{20} = .45$		9	$= .45$
8		$\frac{8}{20} = \frac{8.00}{20} = .4$		8	$= .4$
7		$\frac{7}{20} = \frac{7.00}{20} = .35$		7	$= .35$
6		$\frac{6}{20} = \frac{6.00}{20} = .3$		6	$= .3$
5		$\frac{5}{20} = \frac{5.00}{20} = .25$		5	$= .25$
4		$\frac{4}{20} = \frac{4.00}{20} = .2$		4	$= .2$
3		$\frac{3}{20} = \frac{3.00}{20} = .15$		3	$= .15$
2		$\frac{2}{20} = \frac{2.00}{20} = .1$		2	$= .1$
1		$\frac{1}{20} = \frac{1.00}{20} = .05$		1	$= .05$

And by this Method may any other Table be made.

## S C H O L I U M.

'Tis observed when a Fraction is reduced to the smallest Terms; that if its *Denominator* be compounded only of the prime Numbers 2 and 5 (the Components of 10) the Decimal of that Fraction will be Determined.

But if the *Denominator* be compounded of any other prime Numbers, it will be Indetermined; and the same Figures will return again in Order, and continue to circulate, either by one Figure, or by two, three, &c. Figures, tho' never by more than the Number of Units in the *Denominator* less 1.

For the *Remainder* being always less than the *Divisor*, therefore may be any Number less by 1 than it.

But in so many Operations, at most, as there are Units in the *Divisor*, one of the *Remainders* must return again.

There.

Therefore, the same Figure in the *Quotient* must also return, and so continue the *Circulation*.

*To find the Number of the circulating Figures.*

The most accurate and incomparable Mr *Jones* gives these Observations.

1. Divide the Denominator by 2 and 5 as often as possible; if it come to be 99, 999, 9999, &c. or an *aliquot Part* of such Number, or a Number compounded of 2 or 5, and such aliquot Part; then the Number of the circulating Figures will be equal to so many Figures of 9; as there are in the Number found.

2. If one of the prime Numbers compounding the Denominator (excluding those of 2 and 5) be not an *aliquot Part* of the other; then the Number of the circulating Figures will be equal to the Product of them required by those compounding Prime Numbers.

3. And when the Denominator is compounded of 2, or 5, or any Power of them; then the circulating Figures begin at such a Place of Decimals, as is denoted by the Index of 2 or 5, assumed in that composition, more 1.

### PROPOSITION.

**T**O find the Value of a Decimal Fraction.

This is but converse of the former, and therefore the Rule for finding the true Value of a Decimal is grounded upon the same Reason, as that for turning a Number into a Decimal.

For it will hold, as the Decimal Denominator is to its Numerator, so are the Parts of the next inferior Denominator to the Numerator or Number of such Parts contained in the Decimal. And hence comes this

### R U L E.

*Multiply your given Decimal by the Parts of the next inferior Denominator that is equal to the Integer, the Decimal gives the Part thereof; and if there be any Remainder, by the next inferior Denomination, &c. till all is done.*

This will be rendered very familiar by an Example or two,

## EXAMPLE I.

What is the Value of .95 of a Pound?

Thus  $.95 \times 20s. = 19.00s.$  Answer 19s.

## EXAMPLE II.

What is the Value of .125 of a Pound?

Thus  $.125 \times 20 = 2.500$ ,  $.500 \times 12 = 6d.$  Answer 2s. 6d.

## EXAMPLE III.

What is the Value of .8546 of a Pound?

Thus  $.8546 \times 20 = 17.0920$ ,  $.0920 \times 12 = 1.104$ .  
So 17s. 1d. .104 = .8546; and so of any other.

## EXAMPLE IV.

What is the Value of .54361 of a Tun Weight?

Thus  $.54361 \times 20 = 10.87220$ ,  $.87220 \times 4 = 3.48880$   
 $\times 28 = 13.68640$ ,  $.6864 \times 16 = 10.9824$ . Therefore  
 $.54361 = 10C. 3qr. 13lb. 10dr.$

## EXAMPLE V.

What is the Value of .9434 of an Ounce Troy?

Thus  $.9434 \times 20 = 18.8680 \times 24 = 20.832$ .  
So  $.9434 vz. = 18pw. 20gr.$





# ADDITION.

## PROPOSITION I.

To add Decimal Fractions.

### RULE.

*Whether the Numbers given be pure or mixed Decimals, or some of them whole Numbers, write them down under one another, in such order, that the decimal Points on the left stand all in a line, or under one another; and the Figures all in distinct Columns, in order as they are removed from the Point either on the Right or Left: Then, beginning at the Column on the right Hand, add the Figures in every Column together, just as you do in whole Numbers, placing a Point in the Sum under the Points of the given Numbers.*

### EXAMPLES

$$\begin{array}{r}
 \text{l.} \\
 54.95 \\
 37.05 \\
 36.8 \\
 21.75 \\
 37.5 \\
 \hline
 \text{£. } 188.05 \\
 \text{Or } 188 \quad \text{I} \quad 0
 \end{array}$$

$$\begin{array}{r}
 \text{l.} \\
 3741.8196 \\
 514.5 \\
 91.76184 \\
 9841.546 \\
 \hline
 51.95 \\
 \hline
 14241.57744 \\
 \hline
 14241 \quad \text{II} \quad 6\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{C.} \\
 541. \\
 1.974 \\
 .568 \\
 .037 \\
 .009 \\
 \hline
 543.588 \\
 \hline
 \text{C. } 543 \quad 3 \quad 9
 \end{array}$$

$$\begin{array}{r}
 \text{l.} \quad \text{s.} \\
 .371 \\
 .25 \\
 .8501 \\
 .384 \\
 \hline
 58. \\
 \hline
 59.8551 \\
 \hline
 \text{£. } 59 \quad 17 \quad 1
 \end{array}$$



# SUBTRACTION.

## PROPOSITION II.

*To subtract Decimal Fractions.*

**T**HIS is to be done in the same way as Subtraction in whole Numbers, only observe that you place every Figure under that of the like Name as *Addition*, then subtract.

### EXAMPLES.

From 584.164  
Take 396.896  
Remains 187.268

From 496 take .851  
Thus  $\left\{ \begin{array}{l} 496.000 \\ \quad .851 \\ \hline 495.149 \end{array} \right.$

From 851 take .9849  
Thus  $\left\{ \begin{array}{l} 851. \\ \quad .9849 \\ \hline 850.0151 \end{array} \right.$   
Remains 850.0151

From 681.01  
Take 596.461  
Remains 84.549

From 81.913  
Take 79.04963  
Remains 2.86337

From 11.1101  
Take 9.999999  
Remains 1.110101



# PROPOSITION III. MULTIPLICATION

*Of Decimal Fractions.*

## R U L E.

**M**ultiply the Factors as if all were Integers; and the Decimals in the Product must be equal to the Sum of those in both Factors; if they are not, prefix Cyphers to supply the Defect.

For the Index of each Figure in the Product must be equal to the Sum of the Indices of the multiplied and multiplying Figures.

Thus, Mult. 3.52 by 4.3, the Product is 15.136.

For  $\left\{ \begin{array}{l} 3.52 = 1^{\frac{52}{100}} \\ 4.3 = 4^{\frac{3}{10}} \end{array} \right\}$  and  $1^{\frac{52}{100}} \times 4^{\frac{3}{10}} = 1^{\frac{523}{1000}} = 15.136$ .

Also .013 mult. by .005, gives .000065.

For  $\left\{ \begin{array}{l} .013 = 1^{\frac{13}{1000}} \\ .005 = 1^{\frac{5}{1000}} \end{array} \right\}$  and  $1^{\frac{13}{1000}} \times 1^{\frac{5}{1000}} = 1^{\frac{65}{100000}} = .000065$ .

## S C H O L I U M

When a decimal or mixt Number is to be multiplied by 10, 100, 1000, 10000, &c. 'Tis only removing the separating Point in the Multiplicand so many Places towards the right Hand, as there are Cyphers in the Multiplier.

.8546 mult. by  $\left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \end{array} \right\}$  Product  $\left\{ \begin{array}{l} 8.546 \\ 85.46 \\ 854.6 \\ 8546 \end{array} \right\}$

### EXAMPLE I.

Mult. 38.546  
By 8.92  
Product 343.83032

### EXAMPLE II.

Mult. .54197  
By 452  
Product 244.97044

## EXAMPLE III.

$$\begin{array}{r} \text{Mult.} \quad .2349 \\ \text{By} \quad .516 \\ \hline \text{Product} \quad .1212084 \end{array}$$

## EXAMPLE IV.

$$\begin{array}{r} \text{Mult.} \quad .154372 \\ \text{By} \quad .2168 \\ \hline \text{Product} \quad .0333678496 \end{array}$$

## EXAMPLE V.

$$\begin{array}{r} \text{Mult.} \quad .005468 \\ \text{By} \quad .0541 \\ \hline \text{Product} \quad .0002958188 \end{array}$$

## EXAMPLE VI.

$$\begin{array}{r} \text{Mult.} \quad .54198 \\ \text{By} \quad .05416 \\ \hline \text{Product} \quad .0293536368 \end{array}$$

## S C H O L I U M.

In *Multiplication*, if it were required to find only an assigned Part of the Product ;

1. *Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whose Place you intend to keep in that Product, then invert the Order of all the other Figures, that is, write them in a contrary way.*

2. *Then in multiplying, always begin at that Figure in the Multiplicand which stands over the Figures you are then multiplying withal, and set down the first Figure of each particular Product, directly one under another.*

3. *A due Regard must be had to the Increase, arising from the Figure on the right Hand of that Figure in the Multiplicand, which you begin to multiply at.*

## EXAMPLES.

Mult. 9.58 by 3.65.

$$\begin{array}{r} \text{Thus} \quad 9.58 \\ \text{Inverted} \quad 56.3 \\ \hline 2874 \\ 574 \\ 48 \\ \hline 34.96 \end{array}$$

$$\begin{array}{r} 9.58 \} \text{Common} \\ 3.65 \} \text{way.} \\ \hline \text{Prod.} \quad 34.9670 \end{array}$$

Mult.

Mult. 846.938 by 69.939.

$$\begin{array}{r}
 \text{Thus} \quad 846.938 \\
 \text{Inverted} \quad 939.96 \\
 \hline
 5081628 \\
 762245 \\
 76224 \\
 2540 \\
 762 \\
 \hline
 \text{Product} \quad 59233.99
 \end{array}$$

Multiply 3.141592 by 52.7438, for four Places of Decimals.

$$\begin{array}{r}
 \text{Thus} \quad 3.141592 \\
 \text{Inverted} \quad 52.7438 \\
 \hline
 1570796 \\
 62832 \\
 21991 \\
 1257 \\
 94 \\
 25 \\
 \hline
 \text{Product} \quad 165.6995
 \end{array}$$

Multiply .234564 by .725234, for six, five, four Figures of the Product.

$$\begin{array}{r}
 \text{Inverted} \quad .234564 \\
 \hline
 .432527 \\
 .164194 \\
 4691 \\
 1172 \\
 47 \\
 7 \\
 \hline
 \text{Product} \quad .170111
 \end{array}$$

# 80 Of DECIMAL FRACTIONS.

$$\begin{array}{r} \text{Inverted } .234564 \\ \underline{.432527} \end{array}$$

16421

469

117

4

$$\text{Product } \underline{\underline{.17011}}$$

$$\begin{array}{r} .234564 \\ \underline{.432527} \end{array}$$

1642

47

12

$$\text{Product } \underline{\underline{.1701}}$$

Multiply 104226.8672 by .261799388 for four Decimals.

$$\begin{array}{r} \text{Thus } 104226.8672 \\ \underline{.883997162} \end{array}$$

108453734

62536120

1042268

729587

93803

9379

312

83

8

$$\text{Product } \underline{\underline{27286.5294}}$$

Multiply 419.3 by .6375 for the Integers only.

419.3

5736.0

251

13

3

$$\underline{\underline{267}} \text{ required.}$$

# Of DECIMAL FRACTIONS. 81

In the Product of 798.0625 by 78.54 for the Integers only.

$$\begin{array}{r}
 798.0625 \\
 45.87 \\
 \hline
 55864 \\
 6384 \\
 399 \\
 31 \\
 \hline
 62679
 \end{array}$$

The *Reason* of this *Contraction* is obvious.

For the *Index* of the *right-hand Figure* of any *Product* is the Sum of the *Indices* of the *Factors*.

And by inverting the Position of the Figures (as the Rule directs) the Sums of the Indices of each corresponding Place in the *Factors* will be equal among themselves, and therefore equal to the *Index* of the *right-hand Place* of the *Product* required.

But *Products*, whose *Indices* are equal, belong to the same *Place*; therefore must be set under each other, and their Sum must be the *Product* required.

These *Contractions* are of great Use and Facility in multiplying large Numbers by one another, also in resolving *affected Equations*, or the calculating *Problems* in *Trigonometry* by the *natural Sines* and *Tangents*, &c. and in a vast variety of other Branches of Mathematics.



# PROPOSITION IV.

## DIVISION

*Of Decimal Fractions.*

### RULE.

**D**IVIDE as if all were Integers (annexing Cyphers to the Dividend, if need be;) And

Let the first Figure in the Quotient be of the same Name (i. e. have the same Index) with that Figure of the Dividend, which stands (or is imagin'd to stand) over the Unit Place of the Divisor.

For the Index of each Figure in the Quotient must be equal to the Index of the divided Figure, less by the Index of the dividing Figure.

Or the Decimal Places in the Divisor and Quotient must be equal to those in the Dividend; if they are not, prefix Cyphers to the Quotient to supply the Defect.

For the Dividend is equal to the Product of the Divisor and Quotient; but both Factors contain as many Decimal Places, as the Product does:

Therefore, what Decimal Places are in the Dividend more than in the Divisor, must be supplied in the Quotient.

Thus .0416 divided by .26 gives .16 in the Quotient.

For  $\left\{ \begin{array}{l} .0416 = \frac{416}{10000} \\ .26 = \frac{26}{100} \end{array} \right\}$ , and  $\frac{26}{100} \cdot \frac{416}{10000} (= \frac{41600}{260000} = .16$ .

### SCHOLIUM I.

If the Divisor be greater, or have more Decimal Places, than the Dividend; then by annexing Cyphers to the Dividend, the Quotient may be had to any accuracy.

Thus .25).07864.00(.31456.

### SCHOLIUM II.

Therefore, when there is a Remainder after Division, (tho'



(tho' neither Dividend or Divisor consist of any Decimals) it is but adding Cyphers to the Dividend, and proceed to any Exactness.

## S C H O L I U M III.

When a decimal or mixed Number is to be divided by an Unit with Cyphers, it is but removing the Point, or Comma, in the Dividend, so many Places further towards the left Hand, as there are Cyphers annexed to the Unit, prefixing Cyphers to the Dividend to supply vacancy, if need be.

Thus,

468.3, divided by	{	10	{	46.83
		100		4.683
		1000		.4683
		10000		.04683
		100000		.004683

Quotient is

## S C H O L I U M IV.

In dividing by an infinite Number, the Division may oftentimes be very usually contracted, by

Taking as many of the left-hand Figures of the Divisor, as you think convenient, for the first Divisor, by which divide the given Number, and omit one Figure of the Divisor at each following Operation.

$$46876)9876403217(210692.$$

$$468.76)9876403217.00(21069210.7 \&c.$$

Here, by adding as many Cyphers to the Dividend as are Decimals in the Divisor, the Quotient will be whole Numbers, and if any Remainder, add Cyphers, the rest are Decimals.

$$.46876)9876403217.00000(21069210710 \&c.$$

Here I annex 5 Cyphers, as are the Decimals in the Divisor, and the Quotient is whole Numbers; if any Remainder, by adding Cyphers, the rest are Decimals.

$$468.76$$

# 84 Of DECIMAL FRACTIONS.

$$468.76)98764.03217(210.692$$

$$.46876)98.76403217(210.692$$

$$46876).9876403217(.0000210692$$

$$468.76).9876403217(.00210692$$

$$.46876).9876403217(2.10692$$

I shall conclude Division with inserting only some few more Examples, wherein are contained all the Varieties that can happen in *Division of Decimals*.

$$246)160.884(.654$$

$$246)16.0884(.0654$$

$$2.46)1608.84(654$$

$$2.46)16.0884(6.54$$

$$2.46)160884.00(65400$$

$$.0246)160.8840(6540$$

$$.0246).160884(6.54$$

## PROPOSITION V.

**T**O reduce a Decimal into a Common Fraction.

### R U L E.

*Multiply the Numerator of the Decimal by the Denominator of the Common Fraction.*

Thus .875 of a Pound is 17s. 6d.

$$\begin{array}{r} .875 \quad 20 \\ s. \quad 17 \overline{) 500} \end{array} \text{ For } 1000 : .875 :: 20s. : 17s. \frac{5}{8}.$$

$$\begin{array}{r} \quad \quad 12 \\ d. \quad 6 \overline{) 000} \end{array} \text{ And } 10 : .5 :: 12d. : 6d.$$

PROPOSITION VI.

**T**O find a Multiplier that shall effect the same as a given Divisor.

CASE I.

If the Divisor be an Integer, make it a Denominator, and 1 the Numerator of that Fraction shall be the Multiplier required.

$$\text{For } \frac{N}{D} = N \times \frac{1}{D} \text{ or } \frac{3}{4} = 3 \times \frac{1}{4}.$$

CASE II.

If the Divisor be a Fraction,

Its reverse Fraction shall be the Multiplier required.

$$\text{For } \left(\frac{n}{d}\right) \frac{N}{D} \left( = \frac{d}{n} \times \frac{N}{D}, \text{ or } \left(\frac{3}{4}\right) \frac{2}{3} \left( = \frac{4}{3} \times \frac{2}{3} \right).$$

*Note, Hence, Divisor given : 1 :: 1 : Multiplier required.*

And by finding a Factor, when a Divisor is given; or a Divisor, when a Factor is given; one may advantageously contract several tedious Operations in *Arithmetic*, and find variety of excellent useful Rules for Ease and Expedition in *Common and Practical Accounts*.





## EXTRACTION of ROOTS.

**A**NY Number multiplied into itself is said to be squared, or the Square of that Number; *Example*,  $4 \times 4 = 16$ , the Square of 4, or 4 squared. Also the Square of 12 is 144, for  $12 \times 12 = 144$ .

When a Number is given to be extracted for the Square Root, Sir *Isaac Newton* gives this

### R U L E.

*Let it be first noted (says he) with Points in every other Place, beginning from Unity; then write down such a Figure for the Quotient, or Root, whose Square shall be equal to, or nearest less than the Figure or Figures to the first Point. And then subtracting that Square, the other Figures of the Root will be found one by one, by dividing the Remainder by the Double of the Root, as far as extracted, and each Time taking from that Remainder the Square of the Figure that last came out, and the Decuple of the aforesaid Divisor augmented by that Divisor.*

### E X A M P L E S.

Extract the Square Root of 219024.

Thus 219024 (468 the Root.

$$\begin{array}{r}
 16 \\
 86 \overline{) 590} \\
 \underline{516} \\
 928 \overline{) 7424} \\
 \underline{7424} \\
 0
 \end{array}$$

The Operation is thus performed: Find a Figure whose Square shall be equal to, or the next less Square to 21, which I find to be 4; then  $4 \times 4 = 16$ , which sub-

tracted

tracted from 21 leaves 5 ; then bring down 90, the next Point, which being set on the right Hand of the 5, they make together 590 ; now to obtain the next Figure of the Root, you must divide this 590 by 8, the Double of the first Figure in the Quotient, saying, How oft is 8 contained in 59 ? You will find it to be 6 Times ; wherefore place this 6 in the Quotient (being the second Figure of the Root) and place it on the right Hand of the Divisor, which makes it 86 ; then  $86 \times 6 = 516$ , which subtracted from 590 leaves 74, to which add the next Point 24, and it makes 7424, which you must divide by 92, the Double of the Root 46, saying, How many Times is 92 contained in 742 ? and you will find it to be 8 Times ; then  $928 \times 8 = 7424$ , which subtracted will leave no Remainder ; so the true Root is 468, as is easily proved by multiplying it by itself.

For  $468 \times 468 = 219024$ , the proposed Number.

What is the Square Root of 29506624 ?

Thus  $29506624 (5432 = \text{the Root.}$

$$\begin{array}{r}
 25 \\
 \hline
 104 \overline{)450} \\
 \underline{416} \\
 1083 \overline{)3466} \\
 \underline{3249} \\
 10862 \overline{)21724} \\
 \underline{21724} \\
 \dots
 \end{array}$$

Now, according to what I have said above, *viz.* when any Number is multiplied by itself, such *Product* thereby obtained is said to be the *Square* of that Number ; and that Number, from whence such Square arises, is said to be the *Square Root* thereof ; Therefore the Root multiplied into itself, will produce the Number proposed.

For  $5432 \times 5432 = 29506624$ , the proposed Number.

But it is impossible to extract the Root out of every proposed Number, for, there are Numbers *infinite*, which may be proposed, whose Square Roots cannot exactly

actly be expressed in finite Numbers; and such are said to be *surd*, or *irrational*.

To extract the Root of such a Number, add two Cyphers to the Remainder, double the Root, and proceed as before, continuing to add to every Remainder two *Cyphers*, till you get 4 or 5 Places of *Décimals* in the Root, and so the Work may be carried on to any Degree of Nearness.

What is the Square Root of 26174983434?

Thus 26174983434 (161786.84 the Root.

$$\begin{array}{r}
 \text{I} \\
 26 \overline{) 161} \\
 \underline{156} \\
 321 \overline{) 574} \\
 \underline{321} \\
 3227 \overline{) 25398} \\
 \underline{22589} \\
 32348 \overline{) 280934} \\
 \underline{258784} \\
 323566 \overline{) 2215034} \\
 \underline{1941396} \\
 3235728 \overline{) 27363800} \\
 \underline{25885824} \\
 32357364 \overline{) 147797600} \\
 \underline{129429456} \\
 \underline{18368144}
 \end{array}$$

But since the Product  $6 \times 32356$ , or 1941396, subtracted from 2215034, leaves 273638, it is a sign that the Number 161786 is not the Root of the proposed Number 26174983434 precisely, but that it is a little less. And in this Case, and in others like it, if you desire the Root should approach nearer, you must carry on the Operation in *Décimals*, by adding to the Remainder two *Cyphers* in each Operation. Thus the Remainder 273638, having two *Cyphers* added to it, becomes 27363800;

27363800; by the Division whereof by the Double of 161786, or 323572, you will have the first Decimal Figure 8. Then having writ 8 in the Quotient, subtract  $8 \times 3235728$ , or 25885824, from 27363800, and there will remain 1477976; and so by adding two Cyphers more, the Work may be carried on at pleasure.

When the proposed Number is a mixed one, point the Integers as above, and the Decimal Places, beginning in the second Place from the left-hand. An Example or two will make this familiar.

## EXAMPLE.

What is the Square Root of 5479.32641?

First, add a Cypher to the Decimal Places, and it will make three compleat Points. Thus

5479.326410 (74.022 = the Root.

$$\begin{array}{r}
 49 \\
 144 \overline{) 579} \\
 \underline{576} \\
 14802 \overline{) 33264} \\
 \underline{29604} \\
 148042 \overline{) 366010} \\
 \underline{296084} \\
 \underline{69926}
 \end{array}$$

## SCHOLIUM.

As many Points as the Number given will admit of, so many Figures will the Square Root sought consist of.

What is the Square Root of 51?

First annex Cyphers at pleasure, and it will be 51.0000000000.

## EXTRACTION of ROOTS.

Then 51.0000000000(7.14212 the Root.

$$\begin{array}{r}
 49 \\
 141 \overline{)200} \\
 \underline{141} \\
 1424 \overline{)5900} \\
 \underline{5696} \\
 14282 \overline{)30400} \\
 \underline{28564} \\
 142841 \overline{)183600} \\
 \underline{142841} \\
 1428422 \overline{)4075900} \\
 \underline{2856844} \\
 \underline{1219056}
 \end{array}$$

*To extract the Square Root of a Vulgar Fraction.*

First, Extract the Square Root of the Numerator, and then of the Denominator; if there be no Remainder, set the Numerator over the Denominator so extracted, and the Thing is done.

The Square Root of  $\frac{25}{36}$  is  $\frac{5}{6}$ ;

For 25(5 Numerator      36(6 Denominator.

$$\begin{array}{r}
 25 \\
 \hline
 36
 \end{array}$$

Therefore  $\frac{5}{6}$  is the Root required.

But when you cannot extract the Root of a Vulgar Fraction without a Remainder, put the Fraction into a Decimal, and extract the Root thereof, as above.

What is the Square Root of  $\frac{29}{37}$ ?

Thus  $\frac{29.000000}{37} = .783778$ .

And .783778(.826 the Root.

$$\begin{array}{r}
 64 \\
 162 \overline{)437} \\
 \underline{324} \\
 1646 \overline{)11378} \\
 \underline{9876} \\
 \underline{1502}
 \end{array}$$

But



But when the Root is carried on half way, or above, the rest of the Figures may be obtained by Division alone ; as in this Example, if you had a mind to extract the Root to 5 Figures, after the three former .826 are extracted, the two latter may be had by dividing the Remainder by the Double of .826.

And after this manner, if the Root of 46945 was to be extracted to five Places in Numbers ; after the Figures are pointed, write 2 in the Quotient, as being the Figure whose Square, =4, is the greatest contained in 4 (or 4 exactly) the Figure to the first Point ; and having taken the Square of 2 from 4, there will remain 0 ; then having set the two next Figures, viz. 69, see how many times the Double of 2, or 4, is contained in 6 ; and you will see at first sight no more than 1 ; having wrote 1 in the Quotient, and subtracted  $41 \times 1$ , or 41, from 69, there will remain 28 ; and having set down to this the Figures 45, see how many times the Double of 21, or 42, is contained in 284, and you will find 6 ; then write 6 in the Quotient, and having subtracted  $6 \times 426$ , or 2556, there will remain 289. Lastly, to obtain the remaining Figures, divide the Remainder 289 by the Double of 216, or 432, and you will have the Figures .66, which being writ in the Quotient, you will have the Root 216.66.

After the same manner Roots are also extracted out of Decimal Numbers.

Thus 2691.90(51.8835

$$\begin{array}{r}
 25 \\
 \hline
 101 \overline{) 191} \\
 \underline{101} \\
 1028 \overline{) 9090} \\
 \underline{8224} \\
 1036 \overline{) 8660} 835
 \end{array}$$

0.432190(0.65745

$$\begin{array}{r}
 0 \\
 \hline
 43 \\
 \underline{36} \\
 125 \overline{) 721} \\
 \underline{625} \\
 130 \overline{) 969} 745
 \end{array}$$

EXTRACTION *of* ROOTS.

4.3219(2.0789

4.

407)3219

2849

414)3700(89



CUBE

# C U B E   R O O T.

**A**S any Number, or Root, multiplied into itself, produces a Square, or second *Power*, so again that *Product*, or *Square*, multiplied into the same Root, produces a *Cube*, or third *Power*; and that Number from whence such a Cube arises, is the *Cube Root* thereof, &c. as is manifest from the following Table of Powers.

Roots	1	2	3	4	5	6	7	8
Squares	1	4	9	16	25	36	49	64
Cubes	1	8	27	64	125	216	343	512
Biquadrats	1	16	81	256	625	1296	2401	4096
Surfolids	1	32	243	1024	3125	7776	16807	32768
Square cub'd	1	64	729	4096	15625	46656	117649	262144
Second Surfolids	1	128	2187	16384	78125	279936	843543	2097152
Biquadrats squar'd	1	256	6561	65536	390625	1679616	5764801	16777216
Cubes cub'd	1	512	19683	262144	1953125	10077696	40353607	134217728

*Note,*

*Note*, For ease in the following Operations, I make use of this Notation, viz.

$9|^2 = 81$ , or the Square of 9.

$9|^3 = 729$ , or the Cube of 9.

$9|^4 = 6561$ , or 9 raised to the fourth Power; and so of any other Number or Numbers whatever.

The Extraction of the Cubic Root, and of all others, Sir *Isaac Newton* comprehends under this one general

### R U L E.

*Every third Figure beginning from Unity is first of all to be pointed, if the Root to be extracted be a Cubic one; or every fifth, if it be a Quadrato-Cubic, or of the fifth Power, &c. and then such a Figure is to be writ in the Quotient, whose greatest Power (i. e. whose Cube, if it be a Cubic Power, or whose Quadrato-Cube, if it be the fifth Power, &c.) shall either be equal to the Figure or Figures before the first Point, or the next less; and then having subtracted that Power, the next Figure will be found by dividing the Remainder, augmented by the next Figure of the Resolvend, by the next greatest Power of the Quotient, multiplied by the Index of the Power to be extracted; that is, by the triple Square of the Quotient, if the Root be a Cubic one; or by the quintuple Biquadrate, i. e. five times the Biquadrate, if the Root be of the fifth Power, &c. And having again subtracted the greatest Power of the whole Quotient from the first Resolvend, the third Figure will be found by dividing that Remainder, augmented by the next Figure of the Resolvend, by the next greatest Power of the whole Quotient, multiplied by the Index of the Power to be extracted; and so on in infinitum.*

Thus to extract the Cube Root of 13312053, the Number is first to be pointed after this Manner, viz. 13312053. Then you are to write in the Quotient the Figure 2, whose Cube 8 is the next less Cube to the Figures

Subtract the Cube  $13312053(237 \text{ Root})$   
 $8$   
 $2 \times 2 = 4 \times 3 = 12$   $53(4 \text{ or } 3)$   
 Subtract Cube  $12167$   
 $1587)$  Rem.  $11450(7)$   
 Subt. Cube  $13312053$   
 Remains  $0$

To extract the Cube Root of 314432.

For,  $6 \times 6 = 36 \times 3 = 108 \mid 984(8$

And,  $68 \times 68 = 4624 \times 68 = \underline{314432}$

## Of the CUBE ROOT.

To extract the Cube Root of 5735339.

Thus 5735339 (179 Root  
Subtract Cube

$$\begin{array}{r}
 3 \overline{) 47(7} \\
 \underline{4913} \\
 8223(9 \\
 \underline{5735339} \\
 0
 \end{array}$$

$\overline{17}^3 =$   
 $17 \times 17 = 289 \times 3 = 867$   
 $\overline{179}^3 =$

To extract the Cube Root of 32461759.

Thus 32461759 (319 Root  
Subtract Cube

$$\begin{array}{r}
 3 \times 3 = 9 \times 3 = 27 \overline{) 54(1} \\
 \underline{29791} \\
 3) 26707(9 \\
 \underline{32461759} \\
 0
 \end{array}$$

$\overline{31}^3 =$   
 $31 \times 31 = 961 \times 3 = 2883$   
 $\overline{319}^3 =$

To extract the Cube Root of 22069810125.

Thus 22069810125 (2805 Root  
8

$$\begin{array}{r}
 2 \times 2 = 4 \times 3 = 12 \overline{) 140(8} \\
 \underline{21952} \\
 11781(05 \\
 \underline{22069810125} \\
 0
 \end{array}$$

$\overline{28}^3 =$   
 $28 \times 28 = 784 \times 3 = 2352$   
 $\overline{2805}^3 =$

To extract the Cube Root of 25917056.

Thus 25917056 (295.9 Root  
8

$$2 \times 2 = 4 \times 3 = 12 \quad \overline{179(9}$$

$$29 \overline{)}^3 = \quad 24389$$

$$29 \overline{)}^2 = 841 \times 3 = 2523 \quad \overline{15280(5}$$

$$295 \overline{)}^3 = \quad 25672375$$

$$295 \overline{)}^2 = 87025 \times 3 = 261075 \quad \overline{244681,0 \dots (.9}$$

$$295.9 \overline{)}^3 = \quad 25908060079$$

$$\quad \quad \quad 8995921 \quad \text{Remainder.}$$

And so by adding to the Remainder 3 Cyphers, the Operation may be carried on to any required Places of Decimals.

To extract the Cube Root of 1881365963625.

Thus 1881365963625 (12345 Root.

$$\begin{array}{r} \overline{1} \\ 3 \overline{)} 8 \quad (2 \\ \underline{1728} \end{array}$$

$$\begin{array}{r} 432 \overline{)} 1533 \quad (3 \\ \underline{1860867} \end{array}$$

$$\begin{array}{r} 45387 \overline{)} 204989 \quad (4 \\ \underline{1879080904} \end{array}$$

$$\begin{array}{r} 4568268 \overline{)} 22850596 \quad (5 \\ \underline{1881365963625} \\ \quad \quad \quad 0 \quad 0 \quad 0 \end{array}$$

S C H O L I U M.

1. In Square and Cube Numbers, there are certain Properties worth the Reader's Attention; as, if from an Unit you successively add the odd Numbers, all those Numbers shall be square Numbers; as if to 1, you add the next odd Number, which is 3, the Sum is 4, a Square Number; to which add 5, the next odd Number, and the Sum is 9, a Square Number also; to which

add 7 the next odd Number, and the Sum is 16, a Square Number also, and so on.

2. Likewise, if Cubic Numbers be successively added from Unity, these Numbers will be also Square Numbers.

*To extract the Cube Root of a Vulgar Fraction.*

Reduce the Fraction to its lowest Terms, and extract the Cube Roots of the *Numerator* and *Denominator*, for a new *Numerator* and *Denominator*. But if the *Fraction* be a *Surd*, reduce it to a *Decimal*, always remembering to let your *Decimal Fraction* consist of *Ternaries* of Places, as *three, six, nine, twelve, &c.*

EXAMPLES.

The Cube Root of  $\frac{128}{4}$  is  $=\frac{5}{4}$ .

For  $\frac{128}{4}$  is  $=\frac{27}{4}$ , and the Cube Root of  $\frac{27}{4}$  is  $\frac{3}{4}$ .

The Cube Root of  $\frac{512}{1000}$  is  $=\frac{8}{10}$ .

For  $\frac{512}{1000}$  is  $=\frac{64}{125}$ , and the Cube Root is  $\frac{4}{5}$ ; and so of any other.

To extract the Cube Root of  $\frac{1}{32}$ .

Here the proposed Fraction being a *Surd*, reduce it into a *Decimal Fraction*, which is  $=.358255$ .

Then  $.358255(.71)$ , the Root of  $\frac{1}{32}$ .

$$\begin{array}{r} 343 \\ 147 \overline{) 152(1} \\ 357911 \\ \hline 344 \end{array}$$

Which may be carried on to any Degree of exactness by adding ternary Cyphers.





# BIQUADRATE ROOT.

**A**NY Number involved *four* times produces a *Biquadrate*. Therefore to extract the *Biquadrate* Root of any proposed Number is no more than extracting twice the Square Root, *viz.* the Square Root of the first Root.

## EXAMPLES.

Extract the Biquadrate Root of 4857532416.

Thus 4857532416 (69696

$$\begin{array}{r}
 36 \\
 129 \overline{) 1257} \\
 \underline{1161} \\
 1386 \overline{) 9653} \\
 \underline{8316} \\
 13929 \overline{) 133724} \\
 \underline{125361} \\
 139386 \overline{) 836316} \\
 \underline{836316} \\
 0
 \end{array}$$

And 69696 (264, the Biquadrate Root.

$$\begin{array}{r}
 4 \\
 46 \overline{) 296} \\
 \underline{276} \\
 524 \overline{) 2096} \\
 \underline{2096} \\
 0
 \end{array}$$

For  $264 \times 264 \times 264 \times 264 = 4857532416$ .

To extract the Biquadrate Root of 296637086736.

Thus  $\sqrt{296637086736} = 544644$ .

And  $\sqrt{544644} = 738$  the Biquadrate Root.

*Note,* The above may be expressed thus :

# 100 *Of the BIQUADRATE ROOT.*

$\sqrt{\phantom{x}} : \sqrt{296637086736}$ ; or thus,  $\sqrt{296637086736}^{\frac{1}{2}}$ ;

or thus,  $\sqrt{296637086736}^{\frac{1}{4}} = 738$ .

Also the Biquadrate Root of 99887766554411 is = 3161.38.

For  $\sqrt{99887766554411} = 9994381.7494$ .

And  $\sqrt{9994381.7494} = 3161.38$ .

And so of any other.



QUA-

# QUADRATO-CUBICAL ROOT, or SURSOLID.

ANY Number involved *five* times produces, a Quadrato-Cube, and the Number thereof is the Root.

To extract the Quadrato-Cubical Root of 36430820, it must be pointed over every *fifth* Figure, thus, 36430820 and the Figure 3, whose Quadrato-Cube (or fifth Power) 243 is the next less to 364, *viz.* to the first Point, must be writ in the Quotient; as appears by inspection of the Table. Then the Quadrato-Cube 243 being subtracted from 364, there remains 121, which augmented by the next Figure of the Resolvend, *viz.* 3, and divided by five times the Biquadrate of the Quotient, *viz.* by seeking how many times  $5 \times 81$ , or 405, is contained in 1213, it gives 2 for the second Figure. That Quotient 32, being thrice multiplied by itself, makes the Biquadrate 1048576; and this again multiplied by 32 makes the Quadrato-Cube 33554432, which being subtracted from the Resolvend leaves 2876388. Therefore 32 is the Integer Part of the Root, but not the exact Root; wherefore, if you have a Mind to prosecute the Work in Decimals, the Remainder, augmented by a Cypher, must be divided by five times the aforesaid Biquadrate of the Quotient, by seeking how many times  $5 \times 1048576$ , or 5242880, is contained in 28763880, and there will come out the third Figure, or the first Decimal 5; and so by subtracting the Quadrato-Cube of the Quotient 32.5 from the Resolvend, and dividing the Remainder by five times its Biquadrate, the fourth Figure may be obtained.

And so on *in infinitum.* See Newton's *Arithmetica Universalis.*

# 102 *Of the* QUADRATO-CUBICAL ROOT.

OPERATION.

36430820(32.5

243

405)1213(2

33554432

5242880) 2876388,2(5

To extract the Quadrato or Cubical Root of

102434508843424.

Thus 102434508843424(634

7776

6480)24674(3

992424636

78763860)319204524(4

102434508843424





## Of ALGEBRA.

---

**A**LGEBRA is a Science of universal Quantity, whereby difficult Questions in Arithmetic and Geometry may be solved; and is called the *Analytic Art*, and may be defined to be a *general Method of Reasoning, in which by assuming the Quantity sought as if it were known*, by the help of one or more Quantities really known, we proceed by *Addition, Subtraction, Multiplication, or Division*, till at last the Quantity sought is found equal to some real known Quantity, and so itself comes to be discovered.

In this Art Quantities are represented by Letters, and sometimes by Figures, if there be occasion; and these are either Affirmative or Negative. An affirmative Quantity is denoted by the Sign  $+$ , and is defined to be whatsoever is greater than nothing. A negative Quantity is denoted by the sign  $-$ , and signifies a Quantity less than nothing. The Meaning whereof is, that, whereas all contrary Quantities meet in one common Limit, which equally partakes of both Extrems, and is always represented by a Cypher or 0, if all Quantities on one side the Limit be considered as Affirmative, then all those on the contrary side ought to be look'd upon as Negative. These Signs always belong to the Quantities immediately following them; and all Quantities with these two Signs are always to be interpreted in a contrary Signification: If  $+$  signifies upward, forward, above, before, gain, increase, addition, &c. then  $-$  is to be interpreted, downwards, backwards, below, behind, loss, decrease, subduction, &c. And if  $+$  be understood of these, then  $-$  is to be interpreted of the contrary. And as these affirmative and negative Quantities are contrary to one another in their own Natures, so likewise are they in their Effects; which Consideration

tion alone, if duly attended to, would answer all the Questions concerning the Signs of Quantities, arising from their Addition, Subtraction, Multiplication, &c.

This Art is very deservedly reputed the very Apex of human Learning; for, by means thereof, surprising Truths have been found, as well in natural Philosophy, as pure Mathematics. By this many geometrical Demonstrations are wonderfully abridg'd, and Problems solved, which would be otherwise impossible to be effected; nay even such a Number of Truths is often expressed in one Line by this Art, as would require a whole Volume to expound and demonstrate, otherwise; and by contemplating one Line for a few Hours, you may learn what would take up a whole Year to be learned according to the common Methods.

### NOTATION of ALGEBRA.

1. Letters of the Alphabet may stand for any Number or Quantity whatever, whether known or unknown. Though it is usual to put the first Letters of the Alphabet,  $a, b, c$ , &c. for known Quantities; and the last  $x, y, z$ , &c. for unknown ones. Though some put Vowels  $a, e, y$ , &c. for unknown Quantities.

2. In Operations perform'd by *Literal Arithmetic*, all Quantities, *known*; or *unknown*, are represented by Letters, with prefix'd Marks or Signs, whereby (according to the Nature of the *Proposition*) they may be so ordered by Addition, Subtraction, &c. as if each particular Part was actually known; so that the Position and Relation of one Quantity to another are visible thro' the whole Course of the Process, and consequently that of the *known* to the *unknown*.

3. The Number of times any Quantity is taken must be prefix'd to it, and is called a *Co-efficient*, or *Co-factor*; as  $2a$  denotes 2  $a$ 's,  $5b$  five  $b$ 's,  $25x$  twenty five  $x$ 's.

4. A Quantity having no Sign before it, must always be taken to be Affirmative; and if it hath no numeral Coefficient before it, *Unity* must always be understood; as  $a$  is  $1a$ ,  $x^2$  is  $1x^2$ , &c.

5. *Simple*

5. *Simple Quantities* are those that have but one Member; as  $3b$ ,  $2x$ , &c. *Compound Quantities* are those which are connected by  $+$  and  $-$ ; as  $5a+ba-m-y+g$ .

6. Two or more Species, immediately connected together, denote a *Product*, or Quantity made by the *Multiplication* of all the Species together. Thus  $ab$  denotes a Quantity made by multiplying  $a$  by  $b$ , and  $bmxy$  denotes  $b \times m \times x \times y = bmxy$ ; the Product arising from  $b$  multiplied by  $m$ , and that Product by  $x$ , and the Product again by  $y$ ; but the chief Use of these Notes is, when compound Quantities, as  $m-x$ , were to be multiplied by  $m+3x$ . The Way is to draw a Line over each Quantity, and write them thus:  $\overline{m-x} \times \overline{m+3x}$ . Also  $3a+4b-zx$  to be multiplied by  $mz-zy+mx+10y^2$ , is to be wrote thus:  $\overline{3a+4b-zx} \times \overline{mz-zy+mx+10y^2}$ ; and so of any other.

7. One Quantity below another, with a Line interposed, denotes a *Quotient*, or a Quantity, arising by the Division of the upper Quantity by the lower. As  $\frac{m}{n}$  denotes a Quantity which arises by dividing  $m$  by  $n$ . Likewise thus:  $\frac{xx-yy}{m+r}$  denotes a Quantity arising by dividing  $xx-yy$  by  $m+r$ ; and so in others. Sometimes the Divisor is set before the divided Quantity; as thus:  $\overline{m+r} )xx-yy$ .

8. If a Quantity be multiplied by itself, the Number of *Facts*, or Products, is, for shortness sake, set at the top of the Letter. Thus for  $xx$  we write  $x^2$ ; for  $xxxx$ ,  $x^4$ ; for  $mmmaaabbxx$  we write  $m^3a^3b^2x^2$ . And these Numbers standing above the Letters are called *Indices*; thus 4 is the Index of  $x$  in the Quantity  $x^4$ .

9.  $\frac{2}{3}a^2b^3x$  denotes two thirds of  $a^2b^3x$ , and  $5\frac{a}{b}$  signifies

five times  $\frac{a}{b}$ , and  $11\sqrt{mx}$ , eleven times  $\sqrt{mx}$ . Also

$\frac{m}{n}z$  denotes the Product of  $z$  by  $\frac{m}{n}$ ; and  $\frac{24xx}{14a+by}y^s$  de-

notes the Product made by multiplying  $y^s$  by  $\frac{24xx}{14a+by}$ ,

that is,  $y^s$  multiplied into the Quotient arising by the Di-

vision of  $24xx$  by  $14a+by$ ; and  $\sqrt{\frac{24xx}{14a+by}}y^s$  denotes the

Sq. Root thereof; and  $\sqrt{5a+3x}\sqrt{mx+zy+r^2}$  denotes the

Product of  $\sqrt{5a+3x}$  into the Root  $\sqrt{mx+zy+r^2}$ ; and

$\frac{myz}{\sqrt{\frac{1}{4}aa+bb}}$  denotes the Quantity  $myz$  divided by the Root

$\sqrt{\frac{1}{4}aa+bb}$ ; and  $\sqrt{bb-c}\sqrt{xy+z}$  denotes the Root

$\sqrt{bb-c}$  multiplied into the Root  $\sqrt{xy+z}$ ; and

$\sqrt{xy}\sqrt{mx+z}$  denotes the Root of the Product of the

Quantities  $xy$  into  $\sqrt{mx+z}$ ; and so in other Cases.

And since *Addition*, *Subtraction*, *Multiplication*, and *Division*, are the common Affections of all Quantities, therefore we shall, in the next Place, endeavour, with all the Brevity and Plainness possible, to apply these Rules to *Letters*.

### A X I O M S.

1. If *to* or *from* equal Quantities, equal ones be *added* or *subtracted*, their *Sum* or *Remainder* will be equal.

2. If equal Quantities be *multiplied* or *divided* by equal ones, their *Products* or *Quotients* will be equal.

3. If *from* the Sum of any two Quantities be taken either of them, the *Remainder* will give the other Quantity.

4. The Difference of any two Quantities *added* to the *less* gives the greater Quantity, but *subtracted* from the greater Quantity gives the *less*.

5. The *Product* of any two Quantities divided by either of them, the *Quotient* arising from thence will give the other Quantity.

6. The *Quotient* of any two Quantities being *multiplied* by the *less*, the *Product* is the greater Quantity.

7. Quan-



7. Quantities equal to a Third are equal to one another.

*Note, It would be very proper for the Learner to get these Axioms by heart, as he will find them to be of great use in solving Problems.*



# ADDITION OF ALGEBRAIC INTEGERS.

## C A S E. I.

**W**HEN Quantities have the same Name, and the same Signs ;

### R U L E.

Put down the Sum of their numeral Coefficients, with their common Sign before it, and the common Denomination after it.

### E X A M P L E S.

To	$+a$	$+13b$	$+m$	$-5r$	$xx+yy-2x+5$
Add	$+a$	$+5b$	$+15m$	$-r$	$xx+2yy-3x+12$
Sum	$2a$	$18b$	$16m$	$-6r$	$2xx+3yy-5x+17$

For it is evident from the common Way of numbering, that  $13+5=18$  of any thing of like Name ; as 13 Yards and 5 Yards are 18 Yards.

1	$+a$	$11m$	$n$	$r+s$
2	$+3a$	$19m$	$n$	$r+s$
3	$+5a$	$3m$	$n$	$r+s$
4	$+9a$	$m$	$n$	$r+s$
Sum 5	$18a$	$33m$	$4n$	$4r+4s$

$$\begin{array}{r}
 xz - yu - m + b - 11 \\
 3xz - yu - m + 3b - 19 \\
 2xz - 3yu - 7m + b - 18 \\
 xz - 2yu - m + b - 54 \\
 \hline
 7xz - 7yu - 10m + 6b - 102
 \end{array}$$

## C A S E. II.

When the Quantities have the same Name, and Different Signs,

### R U L E.

Put only the Difference of their Coefficients with the common Denomination after it, and the Sign of the greater Quantity before it.

E x -

EXAMPLES.

To	$+5a$	$-7x$	$-5xy$
Add	$-3a$	$+3x$	$+5xy$
Sum	$+5a-3a$	$-7x+3x$	$+5xy-5xy$
Or,	$+2a$	$-4x$	$0$

To	$11a^2-y+50-x$
Add	$y-40-11a^2+x$
Sum	$11a^2-11a^2-y+y-x+x+50-40$
Or	$10$

For to add a *Negative* is to take away a *Positive*.

Therefore to connect a *Negative* and a *Positive*, is to make the one, and destroy the other.

Thus, if *A.* has 1000*l.* and owes 800*l.* it is evident that the Sum, or his Worth, is but 200*l.*

And if *A.* has 800*l.* and owes 1000*l.* then his Worth is -200*l.* or 200*l.* in debt, or worse than nothing, because he owes 200*l.* more than he actually can pay.

To	$+y$	$-11xy$	$-x$	$za-11+2x+3y-a$
Add	$-3y$	$-21xy$	$+x$	$-3x-5a-za+23-3y$
Add	$-2y$	$+50y$	$-2x$	$-3y+2x-11a+42-3za$
Add	$+4y$	$-15y$	$+2x$	$+3a+4za-2y-21-4x$
	$0$	$+3xy$	$0$	$+za+33-3x-5y-14a$

In ordering such Examples as the above, set down the Quantities having the same Sign, whether affirmative or negative, on a separate Bit of Paper, or a Slate; then gather those of a different Sign, and compare them together, which must be set down according to the Pre-script of the Rule.

Thus	$+za$	$-za$	} It is evident that $+5za$ less $4za$ , the Sum is $+za$ , and so any other.
	$+4za$	$-3za$	
	$+5za$	$-4za$	

C A S E III.

When Quantities are of different Names.

## R U L E.

When the Quantities to be added are of different Denominations, and consequently such as will not incorporate, they can only be put down in one common Series, with their proper Signs before them. So compound Quantities, whose Numbers are all of different Denominations, are incapable of being added any other way, than by being placed one after another without altering the Signs; but if the Members are not all of different Denominations, it may then be convenient to place one compound Quantity under another, with like Parts under like, as far as possible, and then add them together.

## E X A M P L E S.

To	$a$	$3a$	$a+x$	$2a-3x-4y+30$
Add	$b$	$2m$	$b+m$	$24+bb+4a+3x$
Sum	$a+b$	$3a+2m$	$a+x+b+m$	$6a+bb-4y+54$

This sort of Addition is very plain, that  $a$  added to  $b$  cannot be  $ab$ , but  $a+b$ ; for suppose  $a$  stood for 12 Yards, and  $b$  for 6 Shillings, it will neither make 18 Yards, nor 18 Shillings, but 12 Yards *plus* 6 Shillings.

Again, suppose  $a$  denotes 5 Pounds, and  $b$  four Gallons, yet it will not make 9 Pounds, nor 9 Gallons; for 5 Pounds will stand as 5 Pounds, and so will 4 Gallons stand as 4 Gallons; therefore it is manifest they cannot be added together but by connecting the Signs.

Here follow a few Examples promiscuously set to exercise the Learner in the above three Cases of Addition, which, when well understood, he may proceed to Subtraction.

$$\begin{array}{r}
 3y + ab - m + 2z \\
 2y - ab + m + 5zz \\
 15 - 21y + 9m - 3x \\
 \hline
 \text{Sum } 5zz + 2z - 16y + 15 + 9m - 3x
 \end{array}$$

$$\begin{array}{r}
 24+3x+aa-bx+yy \\
 3y+4d+xx+bx-yy \\
 -xx-24-3x+5b-7yy \\
 \hline
 \text{Sum} \quad aa+3y+4d+5b-7yy
 \end{array}$$

$$\begin{array}{r|l}
 aa+2ax+xx & 8by^2+bcx-37y \\
 -4ax & 7by^2-bcx+54y-11 \\
 \hline
 \text{Sum} \quad aa-2ax+xx & 15by^2+17y-11
 \end{array}$$

$$\begin{array}{r}
 5mx+11yz-54xy-54xx-11 \\
 -3mx-11zy+19nx+9zz-11 \\
 -15mx+54xx+17zz+54xy+21 \\
 \hline
 \text{Sum} \quad +6mx+26zz-1
 \end{array}$$





## Subtraction of Algebraic Integers.

**W**HENEVER a simple *Algebraic* Quantity is to be subtracted from another, whether simple or compound, first change the Sign of the Quantity to be subtracted, that is, if it be affirmative, make it, or at least call it, negative, and *vice versa*, and then add it so changed to the other. For since, as was before hinted, the subtracting of any one Quantity from another is the same in effect as adding the contrary, and since changing the Sign of the Quantity to be subtracted renders it contrary to what it was before, it is evident that after such a Change it may be added to the other, and that the Result of this Addition will be the same with that of the intended Subtraction. Thus may the Rule for Subtraction, by changing the Sign of the Quantity to be subtracted, be any time changed into that of Addition, just as the Rule for Division of Fractions, by inverting the Terms of the Divisor, was changed into that of Multiplication.

### EXAMPLES.

From	$+5a$	$+bc$	$-2x$	$11m+12x+13$
Take	$+3a$	$+bc$	$-2x$	$50+11x+29$
Remains	$+5a-3a$	$bc-bc$	$-2x+2x$	$11m+x-66$
Or	$2a$	$0$	$0$	

From	$+5a$	$-12m$	$+9x$	$13xx-12x+20$
Take	$-3a$	$+11m$	$-6x-z$	$xx+x-12$
Remains	$+8a$	$-23m$	$+15x+z$	$12xx-13x+32$

From	$12x+6a-4b-12c^*$	$-7e-5f$
Take	$10x+9a+4b-5c+6d-7e^*$	
Remains	$+2x-3a-8b-7c-6d^*$	$-5f$

## S C H O L I U M.

As to Order, how the several Members or Terms do stand, it matters not in *Addition* and *Subtraction*, so that each has its own peculiar Sign.

For  $a + m - y = a - y + m = -y + m + a = m - y + a = s$ ;

Or  $6 + 8 - 4 = 6 - 4 + 8 = -4 + 8 + 6 = 8 - 4 + 6 = 10$ .



# Multiplication of Algebraic Integers.

**B**EFORE we can proceed to the Multiplication of *Algebraic* Quantities, we are to take notice, that if the Signs of the *Multiplicator* and *Multiplicand* be both alike, that is, both affirmative, or both negative, the Product will be affirmative, otherwise negative.

Thus  $+4$  multiplied into  $+3$ , or  $-4$  into  $-3$ , produces in either Case  $+12$ . But  $-4$  multiplied into  $+3$ , or  $+4$  into  $-3$ , produces in either Case  $-12$ ; all which four Cases are thus demonstrated.

First, that  $+4$  multiplied into  $+3$ , produces  $+12$ , and 2dly, that  $-4$  into  $+3$  gives  $-12$ , is evident from the Nature of Multiplication by an Affirmative, which is but a more compendious Addition. Thirdly, that  $+4$  into  $-3$  gives  $-12$ , is evident from hence, that  $+4$  multiplied into the Progression  $3, 2, 1, 0$ , produces the Progression  $12, 8, 4, 0$ ; therefore  $+4$  multiplied into the rest of the Progressions of Multiplicators, viz.  $-1, -2, -3$ , ought to produce the rest of the Progression of Products, viz.  $-4, -8, -12$ . Fourthly, that  $-4$  multiplied into  $-3$  produces  $+12$  will be evident from the like way of reasoning; for  $-4$  multiplied into the Progression  $3, 2, 1, 0$ , produces the Progression  $-12, -8, -4, 0$ , by the second Case; therefore  $-4$  multiplied into a Continuation of the former Progression, viz.  $-1, -2, -3$ , ought to produce a Continuation of the latter Progression,  $+4, +8, +12$ ; or, more compendiously, thus:  $+4$  into  $+3$  produces  $+12$ , therefore  $+4$  into  $-3$  (which is contrary to  $+3$ ) ought to produce something contrary to  $+12$ , that is,  $-12$ . But if  $+4$  into  $-3$  produces  $-12$ , then  $-4$  into  $-3$  ought to produce something contrary to  $-12$ , namely  $+12$ . And thus the formidable Paradox  $-4$  into  $-3$  produces  $+12$  is found at last to amount to no more than a common Principle in Grammar, viz. *That two Negatives make an Affirmative.*



1.  $+4$  into  $+3$  gives  $+12$
2.  $+4$  into  $-3$  gives  $-12$
3.  $-4$  into  $+3$  gives  $-12$
4.  $-4$  into  $-3$  gives  $+12$ .

Multiply	$4, 4, 4, 4, 4, 4, 4$
Into	$3, 2, 1, *, -1, -2, -3$
	<hr style="border: 0.5px solid black;"/>
	$12, 8, 4, *, -4, -8, -12$

Multiply	$-4, -4, -4, -4, -4, -4, -4$
Into	$+3, +2, +1, *, -1, -2, -3$
	<hr style="border: 0.5px solid black;"/>
	$-12, -8, -4, *, +4, +8, +12$

These Things premised, the Multiplication of simple Algebraic Quantities is performed, first by multiplying their numeral Coefficients together, and then putting down after the Product all the Letters in both Factors, the Sign, when occasion requires, being prefixed as above directed. Thus  $5x \times 4a = 20ax$ .

Tho' this Kind of Language (for it is no more) like all others, be purely arbitrary, yet that a more rational one could not have been invented for this Purpose, will appear by the following Consideration. If any Quantity, as  $x$ , is to be multiplied by any Numbers, as 2, 3, 4, &c. the Product can't be better represented than by  $2x, 3x, 4x$ , &c. Therefore if  $x$  is to be multiplied by  $a$ , the Product ought to be called  $ax$ ; but if  $x$  multiplied into  $a$  produce  $ax$ , then  $4x$  multiplied into  $a$ , ought to produce 4 times as much, that is,  $4ax$ : Lastly if  $4x$  multiplied into  $a$  produce  $4ax$ , then  $4x \times 5a = 20ax =$  five times as much.

*Distinctions to be observed betwixt Addition and Multiplication.*

1.  $a$  added to  $a = 2a$ , but  $ax \ a = aa$
2.  $a$  added to  $*$   $= a$ , but  $ax \ * = *$
3.  $a$  added to  $-a = *$ , but  $ax \ -a = -aa$
4.  $-a$  added to  $-a = -2a$ , but  $-ax \ -a = +aa$
5.  $a$  added to  $1 = a + 1$ , but  $ax \ 1 = a$
6.  $2a$  added to  $-3b = 2a - 3b$ , but  $2ax \ -3b = -6ab$

PROB-

## PROBLEM.

*TO multiply simple Quantities.*

## RULE.

*Join the Factors together, and prefix to them the Product of the Coefficients, if there be any.*

## EXAMPLES.

Multiply	$4bm$	$-gx,$	$-2cd$	$-x$	$-\frac{1}{2}x$
By	$3m$	$4rt$	$15xy$	$-x$	$-\frac{1}{2}x$
Product	$12bm^2$	$-4grtx$	$-30cdxy$	$+xx$	$+\frac{1}{4}xx$

## PROBLEM II.

*TO multiply compound Quantities.*

## RULE.

*Multiply each Part of the Multiplier into each Part of the Multiplicand.*

## EXAMPLES of Compounds by Simples.

Multiply	$a+b$	$x-y$	$mn-41$
By	$m$	$y$	$9$
Product	$ma+mb$	$xy-yy$	$9mn-369$

Multiply	$x-y+mx$
By	$-my$
Product	$-mxy+myy-m^2yx$

## EXAMPLES of Compounds by Compounds.

Multiply	$a+x$	$3y-m+8$
By	$a-x$	$y-9$
	$aa+ax$	$3yy-my+8y$
	$-ax-xx$	$-27y+9m-72$
Product	$aa-xx$	$3yy-my-19y+9m-72$

Multi-

Multiply  $8x^3+5x^2-3x+1$   
By  $x^2-5x+9$

$$\begin{array}{r} 8x^5+5x^4-3x^3+x^2 \\ -40x^4-25x^3+15x^2-5x \\ +72x^3+45x^2-27x+9 \\ \hline \text{Product } 8x^5-35x^4+44x^3+61x^2-32x+9 \end{array}$$

Multiply

$$\begin{array}{r} 3z^2x+myy-zz+24-19x \\ a^7+x^3z-11+21y^5 \end{array}$$

$$\begin{array}{r} 3a^7z^2x+a^7myy-a^7zz+24a^7-19a^7x \\ +3x^4z^3+my^2zx^3-x^3z^3+24x^3z-19x^4z \\ -33z^2x-11my^2+11zz-264+209x \\ +63z^2xy^5+21my^7-21z^2y^5+504y^5-399xy^5 \end{array}$$

Product

$$\begin{array}{r} 3a^7z^2x+a^7myy-a^7z^2+24a^7-19a^7x+3x^4z^3+my^2zx^3-x^3z^3+24x^3z \\ -19x^4z-33z^2x-11my^2+11zz-264+209x+63z^2xy^5+21my^7 \\ -21z^2y^5+504y^5-399xy^5 \end{array}$$

## S C H O L I U M.

Sometimes Products are expressed only by the Quantities to be *multiplied* with the Sign  $\times$  between them.

Thus the Product  $m+x$  by  $n+y$  is  $\overline{m+x \times n+y}$ , and the Product  $a+x$  by  $m-n+y$ , and that Product by  $b+rx$  is  $\overline{a+x \times m-n+y \times b+rx}$ .



# Division of Algebraic Integers.

## General R U L E.

**PUT** the Divisor under the Dividend, with a Line between them.

### E X A M P L E S.

Divide	$a$	$c$	$a^2+m$	$a+dg-m^2$	$m-x+ab$
By	$b$	$x$	$x-g$	$y+z^2$	$r-n+c$
Quotient	$\frac{a}{b}$	$\frac{c}{x}$	$\frac{a^2+m}{x-g}$	$\frac{a+dg-m^2}{y+z^2}$	$\frac{m-x+ab}{r-n+c}$

For, by *common Division*, in dividing 12 by 4, it is the very same Thing whether 3 or  $\frac{12}{4}$  be the Quotient.

The Division of simple Algebraic Quantities (where it is possible) is performed, first, by dividing the numeral Coefficient of the Dividend by the numeral Coefficient of the Divisor, and then putting down after the Quotient all the Letters in the Dividend which were not in the Divisor, the Sign of the Quotient in Division being determined by those of the Divisor and Dividend, just in the same Manner as in Multiplication the Sign of the Product is determined by those of the Multiplier and Multiplicand; that is, if the Signs of the Divisor and Dividend be both *alike*, whether they be both  $+$  or  $-$ , the Quotient will be  $+$ ; otherwise it will be negative. Thus if the Quantity  $-12ab$  is divided by  $-3a$ , the Quotient will be  $+4b$ ; which I thus demonstrate.

In all Divisions whatever, the Quotient ought to be such a Quantity as, being multiplied by the Divisor, will be equal to the Dividend; therefore to enquire for the Quotient in our Case, is nothing else, but to enquire what Number or Quantity multiplied into  $-3a$  the Divisor, will produce  $-12ab$  the Dividend. First then I ask, what Sign multiplied into  $-$ , the Sign of the Divisor, will give  $-$ , the Sign of the Dividend; and the Answer is  $+$ ; therefore  $+$  is the Sign of the Quotient.

tient. In the next place I enquire what Number multiplied into 3, the Coefficient of the Divisor, will give 12, the Coefficient of the Dividend; and the Answer is 4; therefore 4 is the Coefficient of the Quotient. Lastly, I enquire what Letter multiplied into the Letter of the Divisor will produce  $ab$  the Denominator, or literal Part of the Dividend; and the Answer is  $b$ ; therefore  $b$  is the Letter of the Quotient. And thus at last we have the Quotient, which is  $4b$ ; and this Way of reasoning will carry the Learner thro' all the other Cases.

## E X A M P L E S.

Divide	$+mn$	$-mn$	$+mn$	$-mn$	$ax+ay$
By	$+m$	$-m$	$-m$	$+m$	$a$
Quotient	$+n$	$+n$	$-n$	$-n$	$x+y$

For  $n \times m = mn$ , and  $\overline{x+y} \times a = ax+ay$ . Also  $\frac{bx+mx}{m+b}$

is  $=x$ , and  $\frac{ax-ca+nx-cn}{x-c}$  is  $=a+n$ .

## S C H O L I U M I.

If a *Quantity* is found to be a common *Multiplier* in both, it may be expunged from both.

## E X A M P L E S..

Divide	$ab$	$ab+ad$	$ab+abc+abx$	$bcd+cdr$
By	$db$	$acd$	$abd+abf$	$cgd-cmd$
Quotient	$\frac{a}{d}$	$\frac{b+d}{cd}$	$\frac{1+c+x}{d+f}$	$\frac{b+r}{g-m}$

The Division of compound *Algebraic Quantities* is performed, first by ranging the several Members, both of the Divisor and Dividend, according to the Dimensions of some *Letter* common to them both, and then proceeding as in vulgar Arithmetic.

EXAMPLES.

$$\begin{array}{r} 2x+3a) 16x^4+144ax^3+476a^2x^2+684a^3x+360a^4(8x^3+60ax^2+148a^2x+120a^3 \\ \underline{16x^4+24ax^3} \end{array}$$

$$\begin{array}{r} 120ax^3+476a^2x^2 \\ \underline{120ax^3+180a^2x^2} \end{array}$$

$$\begin{array}{r} 296a^2x^2+684a^3x \\ \underline{296a^2x^2+444a^3x} \end{array}$$

$$\begin{array}{r} 240a^3x+360a^4 \\ \underline{240a^3x+360a^4} \end{array}$$

$$\begin{array}{r} 2x+6a) 16x^4+144ax^3+476a^2x^2+684a^3x+360a^4(8x^3+48ax^2+94a^2x+60a^3 \\ \underline{16x^4+48ax^3} \end{array}$$

$$\begin{array}{r} 96ax^3+476a^2x^2 \\ \underline{96ax^3+288a^2x^2} \end{array}$$

$$\begin{array}{r} 188a^2x^2+684a^3x \\ \underline{188a^2x^2+564a^3x} \end{array}$$

$$\begin{array}{r} 120a^3x+360a^4 \\ \underline{120a^3x+360a^4} \end{array}$$

$$3x+4a$$

$$3x+4a)81x^4-108ax^3-495a^2x^2+354a^3x+840a^4(27x^3-72ax^2-69a^2x+216a^3.$$

$$\begin{array}{r} 81x^4+108ax^3 \\ \hline -216ax^3-495a^2x^2 \\ \hline -216ax^3-288a^2x^2 \\ \hline -207a^2x^2+354a^3x \\ -207a^2x^2-276a^3x \\ \hline +630a^3x+840a^4 \\ +630a^3x+840a^4 \\ \hline \end{array}$$

$$3x-7a)81x^4-108ax^3-495a^2x^2+354a^3x+840a^4(27x^3+27ax^2-102a^2x-120a^3$$

$$\begin{array}{r} 81x^4-189ax^3 \\ \hline 81ax^3-495a^2x^2 \\ \hline 81ax^3-189a^2x^2 \\ \hline -306a^2x^2+354a^3x \\ -306a^2x^2+714a^3x \\ \hline -360a^3x+840a^4 \\ -360a^3x+840a^4 \\ \hline \end{array}$$



$$\begin{array}{r}
 2x-3a)16x^4-72a^2x^2+81a^4(8x^3+12ax^2-18a^2x-27a^3) \\
 \underline{16x^4-24ax^3} \\
 +24ax^3-72a^2x^2 \\
 \underline{+24ax^3-36a^2x^2} \\
 -36a^2x^2+54a^3x \\
 \underline{-36a^2x^2+54a^3x} \\
 -54a^3x+81a^4 \\
 \underline{-54a^3x+81a^4} \\
 0
 \end{array}$$

$$\begin{array}{r}
 3x+4a)81x^4-256a^4(27x^3-36ax^2+48a^2x-64a^3) \\
 \underline{81x^4+108ax^3} \\
 -108ax^3-144a^2x^2 \\
 \underline{+144a^2x^2+192a^3x} \\
 -192a^3x-256a^4 \\
 \underline{-192a^3x-256a^4} \\
 0
 \end{array}$$

2a<sup>3</sup>—

$$2a^2 - 3a^2 + a) 6a^{11} - 33a^{10} + 55a^9 - 66a^8 + 66a^7 + 33a^6 - 12a^7 + 8a^6 + 18a^5 - 10a^4 - 24a^3 + 2a^2 + 15a + 5.$$

$$\underline{6a^{11} - a^{10} + 3a^9}$$

$$\underline{-24a^{10} + 52a^9 *}$$

$$\underline{-24a^{10} + 36a^9 - 12a^8}$$

$$\underline{+16a^9 + 12a^8 - 66a^7}$$

$$\underline{16a^9 - 24a^8 + 8a^7}$$

$$\underline{36a^8 - 74a^7}$$

$$\underline{36a^8 - 54a^7 + 18a^6}$$

$$\underline{-20a^7 - 18a^6 + 66a^5}$$

$$\underline{-20a^7 + 30a^6 - 10a^5}$$

$$\underline{-48a^6 - 76a^5 *}$$

$$\underline{-48a^6 + 72a^5 - 24a^4}$$

$$\underline{4a^5 + 24a^4 - 33a^3}$$

$$\underline{4a^5 - 6a^4 + 2a^3}$$

$$\underline{+30a^4 - 35a^3 *}$$

$$\underline{30a^4 - 45a^3 + 15a^2}$$

$$\underline{+10a^3 - 15a^2 + 5a}$$

$$\underline{10a^3 - 15a^2 + 5a}$$

SCHOLIUM II.

When the *Divisor* is not an even Part of the *Dividend*, you may carry on the Operation by annexing to the *Quotient* the Remainder set over the *Divisor*, with a Line drawn between them, in order to terminate it; or else carried on in an *infinite Series*.

EXAMPLES.

$$x+y)zz\left(\frac{zz}{x}-\frac{z^2y}{x^2}+\frac{z^2y^2}{x^3}-\frac{z^2y^3}{x^4}, \text{ \&c. in infinitum}\right.$$

$$\begin{array}{r} zz+\frac{zzy}{x} \\ \hline -\frac{zzy}{x}+0. \\ -\frac{zzy}{x}-\frac{zzy^2}{x^2} \\ \hline +\frac{z^2y^2}{x^2}+0 \\ +\frac{z^2y^2}{x^2}+\frac{z^2y^3}{x^3} \\ \hline -\frac{z^2y^3}{x^3}+0 \\ -\frac{z^2y^3}{x^3}-\frac{z^2y^4}{x^4} \\ \hline +\frac{z^2y^4}{x^4} \end{array}$$

$$\text{For } \frac{x+y}{1} \times \frac{zz}{x} = \frac{z^2x+z^2y}{x} = z^2 + \frac{z^2y}{x}.$$

$$\frac{x+y}{1} \times \frac{z^2y}{x^2} = \frac{z^2xy-z^2y^2}{x^2} = -\frac{z^2y}{x} - \frac{z^2y^2}{x^2}.$$

$$\frac{x+y}{1} \times \frac{z^2y^2}{x^3} = \frac{z^2xy^2+z^2y^3}{x^3} = \frac{z^2y^2}{x^2} + \frac{z^2y^3}{x^3}.$$

$$\frac{x+y}{1} \times \frac{z^2y^3}{x^4} = \frac{-xz^2y^3-z^2y^4}{x^4} = -\frac{z^2y^3}{x^3} - \frac{z^2y^4}{x^4}.$$

## SCHOLIUM III.

The Law of Continuation is manifest, and may be carried on to any Degree of Exactness.

$$y+x)zz\left(\frac{zz}{y}-\frac{z^2x}{y^2}+\frac{z^2x^2}{y^3}-\frac{z^2x^3}{y^4}, \&c. \text{ in infinitum.}\right.$$

$$\begin{array}{r}
 \frac{zz+\frac{zzx}{y}}{\quad} \\
 \frac{-\frac{zzx}{y}}{\quad} + 0 \\
 \frac{\frac{zzx}{y}-\frac{zzx^2}{y^2}}{\quad} \\
 \frac{+\frac{z^2x^2}{y^2}}{\quad} + 0 \\
 \frac{\frac{z^2x^2}{y^2}+\frac{z^2x^3}{y^3}}{\quad} \\
 \frac{-\frac{z^2x^3}{y^3}}{\quad} + 0 \\
 \frac{\frac{z^2x^3}{y^3}+\frac{z^2x^4}{y^4}}{\quad} \\
 \frac{-\frac{z^2x^4}{y^4}}{\quad} \&c.
 \end{array}$$

$$\text{For } \frac{y+x}{1} \times \frac{zz}{y} = \frac{z^2y+z^2x}{y} = z^2 + \frac{z^2x}{y}$$

$$\frac{y+x}{1} \times \frac{z^2x}{y^2} = \frac{z^2xy-z^2x^2}{y^2} = -\frac{z^2x}{y} - \frac{z^2x^2}{y^2}$$

$$\frac{y+x}{1} \times \frac{z^2x^2}{y^3} = \frac{z^2x^2y+z^2x^3}{y^3} = \frac{z^2x^2}{y^2} + \frac{z^2x^3}{y^3}$$

$$\frac{y+x}{1} \times \frac{z^2x^3}{y^4} = \frac{z^2x^3y-z^2x^4}{y^4} = -\frac{z^2x^3}{y^3} - \frac{z^2x^4}{y^4}$$

$1+xx)1(1-x^2+x^4-x^6+x^8 \&c. \text{ in infinitum.}$

$$\begin{array}{r}
 1+xx \\
 \hline
 -xx+0 \\
 \hline
 -xx-x^4 \\
 \hline
 +x^4+0 \\
 \hline
 +x^4+x^6 \\
 \hline
 -x^6+0 \\
 \hline
 -x^6-x^8 \\
 \hline
 +x^8+0 \\
 \hline
 +x^8+x^{10} \\
 \hline
 -x^{10} \&c.
 \end{array}$$

Whence the Law of the Continuation is evident.

$xx+1)1\left(\frac{1}{xx}-\frac{1}{x^4}+\frac{1}{x^6}-\frac{1}{x^8}, \&c. \text{ in infinitum.}\right)$

$$\begin{array}{r}
 1+\frac{1}{xx} \\
 \hline
 -\frac{1}{xx}+0 \\
 \hline
 -\frac{1}{xx}-\frac{1}{x^4} \\
 \hline
 +\frac{1}{x^4}+0, \&c.
 \end{array}$$

For  $\frac{xx+1}{1} \times \frac{1}{xx} = \frac{xx+1}{xx} = 1 + \frac{1}{xx}.$

And  $\frac{xx+1}{1} \times -\frac{1}{x^4} = -\frac{xx+1}{x^4} = -\frac{1}{xx} - \frac{1}{x^4}.$   
 $\&c.$

$xx-yy)xy(y+\frac{y^3}{xx}+\frac{y^5}{x^4}+\frac{y^7}{x^6}+\frac{y^9}{x^8}&c. \text{ in infinitum.}$

$$\begin{array}{r} \frac{xy-y^3}{+y^3+0} \\ \frac{y^3-\frac{y^5}{x^2}}{+\frac{y^5}{x^2}+0} \\ \frac{\frac{y^5}{x^2}-\frac{y^7}{x^4}}{+\frac{y^7}{x^4}, \&c.} \end{array}$$

$xx-yy)xxxz(xz+\frac{y^2z}{x}+\frac{y^4z}{x^3}+\frac{y^6z}{x^5}, \&c. \text{ in infinitum.}$

$$\begin{array}{r} \frac{xxxz-xyz}{+xyz+0} \\ \frac{xyz-\frac{y^4z}{x}}{+\frac{y^4z}{x}+0} \\ \frac{\frac{y^4z}{x}-\frac{y^6z}{x^3}}{+\frac{y^6z}{x^3}+0} \\ \frac{\frac{y^6z}{x^3}-\frac{y^8z}{x^5}}{+\frac{y^8z}{x^5}} \\ \&c. \end{array}$$

*Note,* It is proper to observe that the greatest Term of the Series must stand first; otherwise the Series will not be true, without taking in the Remainder for the Numerator of a Fraction whose Denominator is the Divisor. These six last Examples show how to throw an Expression into an *infinite Series*, which Beginners may omit, till they are better acquainted with Fractions.



# INVOLUTION of QUANTITIES.

## DEFINITION.

*Involution is the multiplying a Root or Number into or by itself, which when done once is called the second Power, or a Square; when that Product is again multiplied by the Root, or original Number, it is called the third Power, or the Cube, &c. Therefore, I say, a Quantity multiplied any number of Times is said to be involved, the Products arising from thence are called Powers, and the Quantity so multiplied a Root.*

$$\text{Thus } \begin{cases} a \times a = aa, \text{ or } a^2. \\ a \times a \times a = aaa, \text{ or } a^3. \\ a \times a \times a \times a = aaaa, \text{ or } a^4. \\ a \times a \times a \times a \times a = aaaaa, \text{ or } a^5. \end{cases}$$

The usual Character or Sign for *Involution* is  $\odot$ .

Quantities, compounded of several Terms, are involved by an actual Multiplication of all their Parts.

Thus to raise  $m+n$  to the fourth Power. See the Operation.

$$\text{First Power} = m+n = \text{Root.}$$

$$\begin{array}{r} m+n \\ m^2+mn \\ +mn+n \end{array}$$

$$\text{Second Power} = m^2+2mn+n^2 = \text{Square, or } (m+n)^2$$

$$\begin{array}{r} m^3+2m^2n+mn^2 \\ +m^2n+2mn^2+n^3 \end{array}$$

$$\text{Third Power} = m^3+3m^2n+3mn^2+n^3 = \text{Cube, or } (m+n)^3$$

$$\begin{array}{r} m^4+3m^3n+3m^2n^2+mn^3 \\ +m^3n+3m^2n^2+3mn^3+n^4 \end{array}$$

$$\text{Fourth Power} = m^4+4m^3n+6m^2n^2+4mn^3+n^4 = (m+n)^4$$

Note,

Note,  $\overline{m+n}^2$  denotes the Square of  $m+n$ .

$\overline{m+n}^3$  denotes  $m+n$  cub'd, or rais'd to the third Power; and is no more than  $m^3 + 3m^2n + 3mn^2 + n^3$  expressed in shorter Terms; and so of any other.

This Method of Notation is now become vastly useful in literal Computations, as solving Problems, &c.

So likewise if it be required to involve  $m-n$  to the fourth Power, the Process will stand thus:

First Power  $= m-n = \text{Root}$

$$\begin{array}{r} m-n \\ mm-mn \\ \hline -mn+nn \end{array}$$

Second Power  $= m^2 - 2mn + n^2 = \text{Square, or } \overline{m-n}^2$ .

$$\begin{array}{r} m-n \\ m^3-2m^2n+mn^2 \\ \hline -m^2n+2mn^2-n^3 \end{array}$$

Third Power  $= m^3 - 3m^2n + 3mn^2 - n^3 = \text{Cube, or } \overline{m-n}^3$

$$\begin{array}{r} m-n \\ m^4-3m^3n+3m^2n^2-mn^3 \\ \hline -m^3n+3m^2n^2-3mn^3+n^4 \end{array}$$

Fourth Power  $= m^4 - 4m^3n + 6m^2n^2 - 4mn^3 + n^4 = \overline{m-n}^4$ , the Power required, and so of any other.

The Method of proceeding is the same in the generating of higher Powers from any given Root, whether *Binomial* or *Multinomial*. As

If  $m+n+a-r$  were to be squared.

Thus.

First Power  $= m+n+a-r = \text{Root}$

$$\begin{array}{r} m+n+a-r \\ m^2+mn+am-rm \\ \hline +mn+nn+an-rn \\ \hline +am+an+aa-ar \\ \hline -rm-rn-ar+rr \end{array}$$

Square  $= m^2 + 2mn + 2am + n^2 + 2an - 2rm - 2rn + aa - 2ar + rr$

Besides



Besides this Method of forming Powers by multiplying the Quantities at length, there is another Rule which performs the Work much shorter, especially in Binomial Quantities; as if  $a+e$  was to be involved to the 7th Power.

R U L E

The Power required must be made to consist of one Term more, than is the Index of the Power required, whereof the first and last are pure Powers, the first of  $a$ , the last of  $e$ , each involved to the Power required; then all the intermediate Terms are to be made up of both  $a$  and  $e$ ; and in these several Terms the Index of  $a$  decreases, and that of  $e$  increases gradually, so that the Sum of their Indices in any Term is equal to the Index of the required Power.

And for finding the Coefficients, or *Unciae*, of the several Terms, the first is always 1, the second is the Index of the Power required. And, in general, if the *Uncia* of any Term be multiplied by the Index of the leading Quantity ( $a$ ), and divided by the Number of Terms to that Place, it gives the *Uncia* of the next following Term.

Thus, to find the 7th Power of  $a+e$ , the Terms without the *Unciae* will be  $a^7, a^6e, a^5e^2, a^4e^3, a^3e^4, a^2e^5, ae^6, e^7$ ; and the particular *Unciae*, or Coefficients, by the Rule- are 1, 7,  $\frac{7 \times 6}{2} = 21$ ,  $\frac{21 \times 5}{3} = 35$ ,  $\frac{35 \times 4}{4} = 35$ ,

$\frac{35 \times 3}{5} = 21$ ,  $\frac{21 \times 2}{6} = 7$ , and  $\frac{7 \times 1}{7} = 1$ . Therefore  $\overline{a+e}^7$   
 $= a^7 + 7a^6e + 21a^5e^2 + 35a^4e^3 + 35a^3e^4 + 21a^2e^5 + 7ae^6 + e^7$ .

And the same Rule holds when the following Quantity ( $e$ ) is negative, only the 2d, 4th, 6th, &c. Terms are negative. Thus

$\overline{a-b}^7 = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7$ .

# EVOLUTION of QUANTITIES.

## DEFINITION.

**E**volution is the extracting Roots out of any Power given, and therefore is called the Analysis of Powers, and is just the Reverse of Involution.

The usual Sign, or Character, for Evolution is  $\sqrt{\phantom{x}}$ .

Thus the Square Root of  $aa$  is  $a$   
 of  $m^2x^2y^2$  is  $mxy$   
 of  $m^4x^4y^4$  is  $m^2x^2y^2$   
 of  $mxy$  is  $\sqrt{mxy}$ , or  $mxy|^{\frac{1}{2}}$

*Note*,  $\sqrt{mxy}$ ,  $mxy|^{\frac{1}{2}}$  are both the same Expression, the former being the old, the latter the new Way of Notation for the Square Root of  $mxy$ , &c.

Also the Cube Root of  $a^3$  is  $a$   
 of  $x^6y^6$  is  $x^2y^2$   
 of  $x^3y^3$  is  $xy$   
 of  $xy$  is  $xy|^{\frac{1}{3}}$  or  $\sqrt[3]{xy}$

The Square Root of  $a$  is  $a^{\frac{1}{2}}$ , or  $\sqrt{a}$ .  
 of  $a+x$  is  $\sqrt{a+x}$ , or  $a+x|^{\frac{1}{2}}$   
 of  $x+\sqrt{mn+y}$  is  $x+\sqrt{mn+y}|^{\frac{1}{2}}$ , or  
 $\sqrt{x+\sqrt{mn+y}}$

Also the Cube Root of  $x$  is  $x^{\frac{1}{3}}$ , or  $\sqrt[3]{x}$ .  
 of  $a+x$  is  $a+x|^{\frac{1}{3}}$ , or  $\sqrt[3]{a+x}$   
 of  $x+\sqrt{mn+y}$  is  $x+\sqrt{mn+y}|^{\frac{1}{3}}$ , or  
 $\sqrt[3]{x+\sqrt{mn+y}}$

Likewise the Biqua-  
 drate Root of  $a^4$  is  $a$   
 of  $x^4y^4$  is  $xy$   
 of  $x$  is  $x^{\frac{1}{4}}$ , or  $\sqrt[4]{x}$

of  $a+x$  is  $\sqrt[4]{a+x}$ , or  $\sqrt[4]{a+x}$ .

of  $xx+aa$  is  $\sqrt[3]{xx+aa}$ .

of  $cx-gy+qq$  is  $\sqrt[3]{cx-gy+qq}$ .

The Square Root of  $49xx$  is  $7x$ .

of  $121x^4$  is  $11x^2$ .

of  $11x^2$  is  $\sqrt{11x^2}$ , or  $x\sqrt{11}$ .

The Cube Root of  $64y^3$  is  $4y$ .

of  $722y^6$  is  $9y^2$ .

of  $119m^3n^2$ , is  $\sqrt[3]{119m^3n^2}$ , or  $\sqrt[3]{119m^3n^2}$ , or  $m\sqrt[3]{119n^2}$ .

*Compound Quantities.*

$aa+2ax+xx(a+x = \text{Root.}$

$$\begin{array}{r} aa \\ 2a) + 2ax + xx \\ \underline{2ax + xx} \end{array}$$

$xx-2ax+aa(x-a = \text{the Root.}$

$$\begin{array}{r} xx \\ 2x-a) - 2ax + aa \\ \underline{-2ax + aa} \end{array}$$

$xx-ax+\frac{1}{4}aa(x-\frac{1}{2}a = \text{Root.}$

$$\begin{array}{r} xx \\ 2x-\frac{1}{2}a) - ax + \frac{1}{4}aa \\ \underline{-ax + \frac{1}{4}aa} \end{array}$$

Here the several Terms are placed in order, of which the given Quantity is composed according to the Demensions of some Letters therein, as you shall think best; and then the Root of the first Term is found, and placed in the Quotient, as in compound Division; and subtracting that Term from the given Quantity, and bringing down the first Term of the Remainder, and

dividing by twice the first Term of the Quotient, or Root, just as the Extraction of the Square Root in whole Numbers; and so repeating the Operation, and dividing the first Term of the Remainder by the same Divisor as in the above Examples, and the Thing is done.

The Roots of higher Powers may sometimes be discovered thus: Extract the Root required out of all the simple Terms which will admit of such a Root; and connect them together with the Signs + and -; which done, involve this Quantity to the same height as the given Power, and if it be the same throughout as the given compound Quantity, you have the Root; if it differ in the Signs, change some of the Signs, and involve it again till they agree.

## EXAMPLE.

To extract the Cube Root of  $a^3 - 6a^2b + 12ab^2 - 8b^3$ ;

here  $\sqrt[3]{a^3}$  is  $=a$ , and  $\sqrt[3]{-8b^3}$  is  $=-2b$ ; therefore I take  $a-2b$  for the Root, which being involved to the third Power, is  $a^3 - 6a^2b + 12ab^2 - 8b^3$ ; therefore  $a-2b$  is the true Root. But if  $a^3 - 6a^2b + 12ab^2 + 8b^3$  was given, I try  $a+2b$ , and it produces  $a^3 + 6a^2b + 12ab^2 + 8b^3$ , which differs from the Quantity given; which has no Root but what is surd, and must be expressed thus,

$$\sqrt[3]{a^3 - 6a^2b + 12ab^2 + 8b^3}; \text{ or thus, } \overline{a^3 - 6a^2b + 12ab^2 + 8b^3}^{\frac{1}{3}}.$$



# ALGEBRAICAL FRACTIONS.

**A** Fraction is a broken Number, expressing some Part or Parts of any thing consider'd as an Integer. It consists of two Quantities, placed one above another, with a Line drawn between them, as in Vulgar Fractions, thus:

Numerators  $\frac{2}{3}$ ,  $\frac{a}{b}$ ,  $\frac{2a-c}{3b}$ ,  $\frac{5bx^2+m-y}{3-y+m}$ , &c.  
 Denominators

## PROPOSITION I.

**T**O Reduce fractional Quantities into a lower Denomination.

### R U L E.

*Divide both Numerator and Denominator by their greatest common Divisor, or by any Quantity that will divide both of them, and the Quotients will be the new Fraction. See p. 18, 19. Vulgar Fractions.*

### E X A M P L E S.

$$\begin{aligned} 1. \frac{mnx}{mxy} &= \frac{n}{y}, & 2. \frac{25z}{5xz+15az} &= \frac{5}{x+3a}. \\ 3. \frac{m^2-n^2}{m^2+2mn+n^2} &= \frac{m-n}{m+n}. \end{aligned}$$

## PROPOSITION II.

**T**O reduce an Integer into an improper Fraction.

### C A S E I.

*When there is no assigned Denominator, then let the given Integer be a Numerator, and Unit its Denominator. See Page 20.*

### E X A M P L E S.

$$x = \frac{x}{1}; \quad m+n = \frac{m+n}{1}; \quad rt = \frac{rt}{1}; \quad x^2+y^2 = \frac{x^2+y^2}{1}.$$

## CASE II.

*When there is an assigned Denominator.*

## R U L E.

*Multiply the Integer by the assigned Denominator, the Product shall be the Numerator.*

## E X A M P L E S.

1. Denom. assign'd      a. Then  $y = \frac{ya}{a}$ ; here the Integer is y.

2. Denom. assign'd      x. Then  $a + b = \frac{ax + bx}{x}$ ; here the Integer is  $a + b$ .

3. Denom. assign'd  $m + n$ . Then  $x = \frac{mx + nx}{m + n}$ ; here the Integer is x.

For  $yx = ya$ ;  $\overline{a + b} \times x = ax + bx$ ;  $x \times \overline{m + n} = mx + nx$ .

## P R O P O S I T I O N III.

*To reduce mixt Fractions into Improper Ones.*

## R U L E.

*Multiply the Integer by the Denominator of the Fraction, and add the Numerator to the Product, subscribing the same Denominator. See Page 21.*

## E X A M P L E S.

$$1. x + \frac{mn}{b} = \frac{bx + mn}{b}.$$

$$2. a + b + \frac{zx}{mr} = \frac{amr + bmr + zx}{mr}.$$

$$3. x + y - cx + my + \frac{xz + yz}{r + c} =$$

$$\frac{rx + ry - rcx + rmy + cx + cy - c^2x + cmy + xz + yz}{r + c}.$$

PROPOSITION IV.

**T**O reduce an improper Fraction into an Integer, or mixt Fraction.

RULE.

The Numerator divided by the Denominator, the Quotient will be the mixt Fraction, or Integer required. See p. 22.

EXAMPLES.

$$1. \frac{bx+mn}{b} = x + \frac{mn}{b}.$$

$$2. \frac{mx+nx}{m+n} = x.$$

$$3. \frac{amr+bmz-az}{mr} = a+b-\frac{zz}{mr}.$$

$$4. \frac{rx+ry-rx+my+cx+cy-c^2x+cmx+xz+yz}{r+c} =$$

$$x+y-\frac{cx+my+zz+yz}{r+c}.$$

PROPOSITION V.

**T**O reduce Fractions of different Denominators into their Equivalents, which shall have the same Denominator.

RULE.

Multiply all the Denominators continually for a common Denominator, and each Numerator continually by the others Denominators for new Numerators, and the Products are the respective Numerators. See Rule, Page 24.

EXAMPLES.

$$1. \frac{a}{b}, \frac{c}{d} \text{ make } \frac{ad}{bd}, \frac{bc}{bd}.$$

$$\text{For } \begin{cases} a \times d = \frac{ad}{bd} \\ b \times d = \frac{bd}{bd} \end{cases} \text{ and } \begin{cases} c \times b = \frac{bc}{bd} \\ d \times b = \frac{bd}{bd} \end{cases}$$

$$2. \frac{y}{x}, \frac{z}{m}, \frac{ab}{c}, \text{ when reduced, make } \frac{cmz}{cmx}, \frac{czx}{cmx}, \frac{abmx}{cmx}.$$

For

$$\left\{ \begin{array}{l} \frac{y \times m \times c}{x \times m \times c} = \frac{c m y}{c m x} (= \frac{y}{x}) \\ \frac{z \times x \times c}{m \times x \times c} = \frac{c z x}{m x c} (= \frac{z}{m}) \\ \frac{a b \times x \times m}{c \times x \times m} = \frac{a b m x}{c x m} (= \frac{a b}{c}) \end{array} \right\}$$





# Addition of Fractions.

## PROPOSITION VI.

**T**O add a Fraction to another, or to subtract one from another.

These are managed as Vulgar Fractions. See p. 33—45.

For when these are reduced to a common Denominator, the Sum or Difference of the Numerators, set over the common Denominator, will be the Sum, or Difference, of the given Fractions.

### EXAMPLES in Addition.

1.  $\frac{a}{b}$ , add to  $\frac{c}{d}$ , is  $= \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ ; by Prop. 5.

2.  $\frac{xx}{m}$ , added to  $\frac{yy}{m}$ , is  $= \frac{xx}{m} + \frac{yy}{m} = \frac{xx+yy}{m}$ .

3.  $\frac{xx+xy}{x-y}$ ; add to  $x$ ,  $= \frac{xx+xy+xx-xy}{x-y} = \frac{2xx}{x-y}$ .

4.  $\frac{xx}{a}$  added to  $\frac{yy}{b}$ , added to  $\frac{zz}{c}$ , is  $= \frac{bcxx+acyy+abzz}{abc}$

For  $\left\{ \begin{array}{l} \frac{xx \times b \times c}{a \times b \times c} = \frac{bcxx}{abc} \\ \frac{yy \times a \times c}{b \times a \times c} = \frac{acyy}{abc} \\ \frac{zz \times a \times b}{c \times a \times b} = \frac{abzz}{abc} \end{array} \right\}$  by Prop. 5.

Therefore  $\frac{bcxx}{abc} + \frac{acyy}{abc} + \frac{abzz}{abc}$  is  $= \frac{bcxx+acyy+abzz}{abc}$ ;

by this Proposition.

5.  $\frac{aa}{m} + \frac{bb}{n} + \frac{cc}{d} + \frac{zz}{xx}$  is  $=$   
 $\frac{andxx+b^2mdx^2+c^2mnx^2+z^2mnd}{mndxx}$

Sub-

# Subtraction of Fractions.

## EXAMPLES in Subtraction.

1. From  $\frac{a}{b}$  take  $\frac{c}{d}$ , the Remainder is  $\frac{ad-bc}{bd}$ .

$$\left. \begin{array}{l} \text{For } \frac{a \times d}{b \times d} = \frac{ad}{bd} \\ \text{And } \frac{c \times b}{d \times b} = \frac{bc}{bd} \end{array} \right\} \text{by Prop. 5.}$$

Therefore  $\frac{ad}{bd} - \frac{bc}{bd}$  is  $= \frac{ad-bc}{bd}$ ; by this Proposition.

2.  $\frac{xx}{m} - \frac{yy}{m} = \frac{xx-yy}{m}$ ; as is evident.

3. From  $\frac{a^4+dx^3}{abd}$  take  $\frac{x^3}{ab}$ , the Remainder is  $= \frac{aaa}{bd}$ .

$$\text{For } \frac{a^4+dx^3}{abd} - \frac{x^3}{ab} = \left( \frac{a^5b^2+abdx^3-abdx^3}{a^2b^2d} \right) = \frac{aaa}{bd}.$$

4. From  $\frac{2xy+y^2}{x+y}$  take  $y$ , the Remainder is  $\frac{xy}{x+y}$ .

$$\text{For } \frac{2xy+y^2}{x+y} - y = \left( \frac{2xy+y^2-xy-y^2}{x+y} \right) = \frac{xy}{x+y}.$$

5. From  $\frac{a^3+n^3}{cx-xx}$  take  $\frac{n^4}{a^2c-a^2x}$ , the Remainder is  $=$

$$\frac{a^5+a^2n^3-n^4x}{a^2cx-a^2x^2}.$$



# PROPOSITION VII. Multiplication of Fractions.

## R U L E.

**F**IRST reduce whole or mixt Quantities into improper Fractions, and then multiply the Numerators for a new Numerator, and the Denominators for a new Denominator. See Page 46, 47, &c.

## E X A M P L E S.

1.  $\frac{a}{b}$  multiplied by  $\frac{c}{d}$  is  $= \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

2.  $\frac{mx}{cy} \times \frac{dy}{cz} = \frac{mdxy}{ccyz}$

3.  $\frac{bx}{c}$  multiplied by  $x$  is  $\frac{bx^2}{c}$ , for  $\frac{bx}{c} \times \frac{x}{1} = \frac{bx^2}{c}$

4.  $\frac{ax}{m} \times a+x$  is  $= \frac{ax}{m} \times \frac{a+x}{1} = \frac{aax+axx}{m}$

5.  $\frac{x^3}{ax+aa}$  multiplied by  $x+a$  is  $= \frac{x^3}{ax+aa} \times \frac{x+a}{1} = \frac{x^4+ax^3}{ax+aa} = \frac{x^3}{a}$

6.  $x + \frac{aa}{x-a}$  multiplied by  $x-a$  is  $= \frac{xx-ax+aa}{x-a} \times x-a = xx-ax+aa$

7.  $x + \frac{aa}{x-a} \times x-2a + \frac{aa}{x}$  is  $= \frac{x^2-ax+aa}{x-a} \times \frac{xx-2ax+aa}{x} = \frac{x^4-3ax^3+4a^2x^2-3a^3x+a^4}{x-ax} = x^2 - 2ax + 2a^2 - \frac{a^3}{x}$

For  $\frac{a^3}{x} \times \frac{xx-ax}{x} = \frac{a^3xx-a^4x}{x} = a^3x-a^4$

If any Fraction be to be multiplied by the Denominator, the Numerator is the Product. PRO-

# PROPOSITION VIII.

## Division of Algebraic Fractions.

**I**S performed juſt in the ſame Way as Vulgar Fractions; invert the Diviſor, and multiply them by the laſt Rule. See page 52, 53, &c.

### EXAMPLES.

1.  $\frac{a}{b}$  divided by  $\frac{x}{y}$  quotes  $\frac{ay}{bx}$ .

For  $\frac{y}{b} \times \frac{a}{y} = \frac{ay}{bx}$ .

2.  $\frac{axy}{bcm}$  divided by  $\frac{r}{s}$  quotes  $\frac{asxy}{bcmr}$ .

For  $\frac{s}{r} \times \frac{axy}{bcm} = \frac{asxy}{bcmr}$ .

3.  $\frac{a+3}{5}$  divided by  $\frac{20x}{3}$  gives  $\frac{3a+9}{100x}$ .

For  $\frac{3}{20x} \times \frac{a+3}{5} = \frac{3a+9}{100x}$ .

4.  $\frac{mx+21}{17}$  divided by  $\frac{11yz}{12}$  gives  $\frac{12mx+252}{187yz}$ .

For  $\frac{12}{11yz} \times \frac{mx+21}{17} = \frac{12mx+252}{187yz}$ .

5.  $\frac{y^3}{aa}$  divided by  $x$  gives  $\frac{y^3}{a^2x}$ .

For  $\frac{1}{x} \times \frac{y^3}{a^2} = \frac{y^3}{a^2x}$ .

6.  $x \div \frac{y^3}{a^2} = \frac{a^2}{y^3} \times x \left( = \frac{a^2x}{y^3} = \frac{a^2}{y^3} \times \frac{x}{1} = \frac{a^2x}{y^3} \right)$ .

7.  $x-y \div \frac{x-y}{a} = \frac{a}{x-y} \times (x-y) \left( = \frac{ax-ay}{x-y} = a \right)$ .

8.  $x^2 +$

$$8. \frac{x^2 + 2ax + a^2 - bx - qb}{ax + bx - ab - bb} \div \frac{x + a - b}{a + b} =$$

$$\frac{a + b}{x + a - b} \left( \frac{x^2 + 2ax + a^2 - bx - ab}{ax + bx - ab - bb} \right) = \frac{x + a}{x - b}$$

The Demonstration to the above may be had thus; an Instance of which take Example first; where it is said that  $\frac{a}{b} \div \frac{x}{y}$  gives  $\frac{ay}{bx}$ .

For  $\frac{a}{b}, \frac{x}{y} = \frac{ay}{by}, \frac{bx}{by}$ ; by Prop. 5.

But  $\frac{ay}{by} : \frac{bx}{by} :: ay : bx$ .

That is,  $\frac{a}{b} : \frac{x}{y} :: ay : bx$ ; by equal Division.

Therefore  $\frac{a}{b} \div \frac{x}{y} = \frac{ay}{bx}$ .

When the Fractions have a common Denominator, divide one Numerator by the other.

Because Fractions, having the same Denominators, are as their Numerators; by Cor. 3. page 14.

### Of Proportional Quantities.

*Ratio*, or *Proportion*, is the Relation which one Quantity has to another in respect of Magnitude; of these Quantities the former is called the *Antecedent*, and the latter the *Consequent*.

The *Quantity* of any *Ratio* is the Quotient of the Antecedent divided by the Consequent, and expresses how many fold the one is of the other. See Def. 31. page 6.

Thus if A, B be two Quantities, A the Antecedent, and B the Consequent, the Ratio of A to B is expressed by  $\frac{A}{B}$ ; as if A be 4, and B 2, then the Ratio of 4 to 2 is  $\frac{4}{2}$ , or 2, that is, double.

Quantities are said to be *proportional*, or to have the same *Proportion* to one another, when one Antecedent con-

contains its Consequent as often as any other Antecedent contains its Consequent. Thus if  $\frac{A}{B} = \frac{C}{D}$ , then the

Quantities  $A, B, C, D$ , are called *Proportionals*, and are said to be *geometrically proportional*; and are written thus,  $A : B :: C : D$ . In Numbers, if  $\frac{8}{4} = \frac{6}{3}$ , then 8 is to 4 as 6 is to 3, or  $8 : 4 :: 6 : 3$ .

If there be three or more Quantities  $A, B, C, D, \&c$  and if  $A : B :: B : C :: C : D, \&c$ . then the Quantities  $A, B, C, D, \&c$ . are said to be in *continual Proportion*, or in *Geometrical Progression*; and are written thus,  $A, B, C, D \div \div$ .

### PROPOSITION IX.

**I**F four Quantities  $A, B, C, D$ , are proportional; the Rectangle of the Means is equal to the Rectangle of the Extremes;  $BC = AD$ .

For by Supposition  $\frac{A}{B} = \frac{C}{D}$ ; then multiplying both by  $B$  (by Ax. 2.)  $\frac{A \times B}{B}$ , or  $A = \frac{CB}{D}$ ; and again multiplying both by  $D$ ,  $A \times D = \frac{CB \times D}{D} = CB$ .

### COROLLARY I.

Hence it follows that if the Product of two Quantities,  $AD$ , be equal to the Product of two other Quantities,  $BC$ , these four Quantities are proportional,  $A : B :: C : D$ .

### COROLLARY II.

Hence if three Quantities are in continual Proportion, the Square of the Mean is equal to the Rectangle of the Extremes; for in this Case  $B = C$ , and  $AD = BC = B^2$  or  $C^2$ .

### COROLLARY III.

Hence also, if the Rectangle of two Quantities be equal

equal to the Square of a third; these three Quantities are in continual Proportion.

*Note, The Learner will find this Proposition to be of great Service in bringing Problems to Equations.*

PROBLEM I.

**T**O turn proportional Quantities into Equations.

R U L E.

*Make the Product of the Means equal to the Product of the Extremes.*

E X A M P L E S.

1. If  $a : b :: c : d$ , then  $ad = bc$ .
2. If  $a : b :: b : c$ , then  $bb = ac$ .
3. If  $c + d : b :: e : a$ , then  $bc = ca + da$ .
4. If  $2b + y : a + 2e :: a : y$ , then  $2by + yy = aa + 2ae$ .

PROBLEM II.

**T**O turn Equations into Analogies or Proportions.

R U L E.

*Divide each Side of the Equation into two such Factors as being multiplied together will produce that side. Then make the two Factors of one side of the Equation the two Means, and the two Factors, on the other side, the two Extremes.*

E X A M P L E S.

1. If  $ad = bc$ , then  $a : b :: c : d$ .
2. If  $ee = pq$ , then  $p : e :: e : q$ .
3. If  $bc = ca + da$ , then  $c + d : b :: c : a$ .
4. If  $aa + 2ae = 2by + yy$ , then  $a : y :: 2b + y : a + 2e$ .
5. If  $bc + bd = da + g$ , then  $b : \sqrt{da + g} :: \sqrt{da + g} : c + d$ ; or thus,  $b : 1 :: da + g : c + d$ .



## PROPOSITION X.

### Reduction of Simple Equations.

**A**N Equation in Algebra is that Equality which always exists between two different Expressions, as between one Number, or Quantity, and one; several, and one; several and several, or between their Sums, Differences, Products, Quotients, Powers and Roots, either all expressed particularly by the common numerical Characters, or universally by the Letters of the Alphabet, or by both these together, and known by this Mark  $=$ . The Use of such Equations is for representing more conveniently and more distinctly the Conditions of Problems, when translated out of common Language into that of *Algebra*. As, for Example, let it be supposed to find a Number with the following Property, viz. that  $\frac{2}{3}$  of it, with 4 over, may amount to the same as  $\frac{7}{12}$  with 9 over. Here putting  $x$  for the unknown Quantity, the Condition of this Problem (when translated out of common Language, into that of *Algebra*) will be represented by the following Equation, viz.  $\frac{2x}{3} + 4 = \frac{7x}{12} + 9$

$\frac{7x}{12} + 9$ . Now since in this Equation, as well as almost all others, arising immediately from the Conditions of Problems themselves, the unknown Quantity  $x$ , must be so ordered, that itself alone possessing one side of the Equation, must be found equal to such as are entirely known on the other, that is, in the present Case, to determine the Value of the unknown Quantity  $x$ , is what is commonly called the *Resolution of an Equation*; for the effecting whereof several Precepts, Axioms and Processes are required, some whereof, to wit, such as most fre-



frequently occur, we shall here put down, the rest we shall take notice of occasionally as they offer themselves.

1. If any Quantity be taken from one side of an Equation, and be placed on the other with a *contrary* Sign, which is commonly called *Transposition*, the two Sides of the Equation will be equal to one another. Thus  $12 - 4 = 8$ ; transpose  $-4$ , and you will have  $12 = 8 + 4 = 12$ ; or  $x - a = b$ ; transpose  $-a$ , and you will have  $x = b + a$ .

2. If when an Equation is to be resolved, Fractions are found on one or both Sides, it must be freed from them by multiplying the whole Equation into the *Denominators* of those Fractions successively. Thus if  $\frac{2x}{3} + 4 = \frac{7x}{12} + 9$ , multiply both sides by 3, and you will

have  $2x + 12 = \frac{7x}{4} + 27$ ; multiply again by 4, and you

will have  $8x + 48 = 7x + 108$ ; transpose  $7x$ , and you will have  $x = 60$ .

3. If, after the Equation is thus reduced to integral Terms, the unknown Quantity be found on both Sides the Equation (which in most Cases happens so) let it be brought by *Transposition* to one and the same Side, *viz.* to that Side which, after *Reduction*, will exhibit it *affirmative*. Thus if  $24x + 144 = 21x + 324$ , transpose  $21x$ , and you will have  $24x - 21x + 144 = 324$ , or rather  $3x + 144 = 324$ .

After this, if any loose known Quantities be found on the same Side with the unknown, let them also be transposed to the other Side of the Equation; as we have here 144, joined to the unknown Quantity above, which transposed is  $3x = 324 - 144$ , or rather  $3x = 180$ .

4. If now the unknown Quantity, has any *Coefficients* before it, divide all by that Coefficient, and the Equation will be resolved. Thus if  $3x = 180$ , divide both Sides by 3, and you will have  $x = \frac{180}{3}$ ; that is,  $x = 60$ .

5. If the unknown Quantity, or any other Quantity, be concerned in every Member of the Equation, divide the whole by so much as is common, and the Equation

will be reduced to a more *simple* one ; thus if  $4x^2=196x$ , divide the whole by  $x$ , and we have  $4x=196$ , whence by the last article  $xx=49$ .

6. If at any time the unknown Quantity arises to the second Power, or Square of itself, then extract the Square Root on both sides of the Equation ; thus  $xx=49$  ; then by extracting the Square Root on both sides, you will have  $x=\sqrt{49}$ , or rather  $x=7$ .

### EXAMPLES for clearing simple Equations.

Suppose this Equation,  $\frac{2x}{3}+4=\frac{7x}{12}+9$ .

Method of Operation.	Step	
	1	$\frac{2x}{3}+4=\frac{7x}{12}+9$ .
$1 \times 3$	2	$2x+12=\frac{7x}{4}+27$ .
$2 \times 12$	3	$24x+144=21x+324$ .
$3-21x$	4	$24x-21x+144=324$ .
$4-144$	5	$3x=324-144$ .
that is	6	$3x=180$ .
$6 \div 3$	7	$x=\frac{180}{3}=60$ .

First here is given this Equation  $\frac{2x}{3}+4=\frac{7x}{12}+9$ , to find the Value of  $x$ . I set down the Number of the Steps (1) in the Margin, and multiplying every Part of the Equation by 3, it produces a new Step (*viz.* 2) and is thus  $2x+12=\frac{7x}{4}+27$ , and the marginal Note is  $1 \times 3$ , denoting the first Step to be multiplied quite through by the Denominator 3. Then I observe my Equation in the 2d Step ; and find another Fraction (*viz.*  $\frac{7x}{12}$ ), by whose Denominator I multiply the whole Equation in the 2d Step, and it produces  $24x+144=21x+324$ , and in the Margin I set  $2 \times 12$ , which makes a third Step. Now

Now having got my Equation out of Fractions, I must so transpose the Terms by *Addition, Subtraction, &c.* till I bring the unknown Quantity on one side of the Equation with an affirmative Value as the 5th Step, which order'd according to the Signs, + and —, gives  $3x = 180$  for the 6th Step; then by *Article 4.* above,  $x$  will be = 60; and so of any other.

Equation.	Steps	
	1	$\frac{x}{6} - 4 = 24 - \frac{x}{8}$ . What is $x =$ ?
$1 \times 6$	2	$x - 24 = 144 - \frac{6x}{8}$ .
$2 \times 8$	3	$8x - 192 = 1152 - 6x$ .
$3 + 6x$	4	$8x + 6x - 192 = 1152$ .
$4 + 192$	5	$14x = 1152 + 192 = 1344$ .
$5 \div 14$	6	$x = \frac{1344}{14} = 96$ .

Given this Equation  $a - \frac{x}{b} = \frac{2x}{c} - b$ ; to find the Value of  $x$ .

Thus	1	$a - \frac{x}{b} = \frac{2x}{c} - b$ .
$1 \times b$	2	$ab - x = \frac{2bx}{c} - bb$ .
$2 \times c$	3	$abc - cx = 2bx - b^2c$ .
$3 + cx$	4	$abc = 2bx + cx - b^2c$ .
$4 + b^2c$	5	$2bx + cx = abc + b^2c$ .
$5 \div 2b + c$	6	$x = \frac{abc + b^2c}{2b + c}$ .

Given this Equation  $56 - \frac{3x}{4} = 48 - \frac{5x}{8}$ ; to find the Value of  $x$ , proceed thus;

Steps			
1		$56 - \frac{3x}{4} = 48 - \frac{5x}{8}$	
$1 \times 4$	2	$224 - 3x = 192 - \frac{20x}{8}$	
$2 \times 8$	3	$1792 - 24x = 1536 - 20x$	
$3 + 24x$	4	$1792 = 1536 - 20x + 24x$	
that is	5	$4x + 1536 = 1792$	
$5 - 1536$	6	$4x = 1792 - 1536 = 256$	
$6 \div 4$	7	$x = \frac{256}{4} = 64$	

Given this Equation  $\frac{7x}{8} - 5 = \frac{9x}{10} - 8$ ; to find the Value of  $x$ . Thus

Steps			
1		$\frac{7x}{8} - 5 = \frac{9x}{10} - 8$	
$1 \times 8$	2	$7x - 40 = \frac{72x}{10} - 64$	
$2 \times 10$	3	$70x - 400 = 72x - 640$	
$3 + 640$	4	$640 + 70x - 400 = 72x$	
$4 - 70x$	5	$640 - 400 = 72x - 70x$	
that is	6	$240 = 2x$	
$6 \div 2$	7	$x = \frac{240}{2} = 120$	

Given this Equation  $\frac{ax}{bc} - a + b = c - \frac{dx + ac}{abc}$ , to find the Value of  $x$ , when  $a=12$ ,  $b=3$ ,  $c=6$ ,  $d=1$ .

Thus	1	$\frac{ax}{bc} - a + b = c - \frac{dx + ac}{abc}$
$1 \times bc$	2	$ax - abc + bbc = bcc - \frac{bcdx + abcc}{abc}$
$2 \times abc$	3	$aabcx - a^2b^2c^2 + ab^3c^2 = ab^2c^3 - bcdx + abcc$
$3 + bcdx$	4	$a^2bcx + bcdx - a^2b^2c^2 + ab^3c^2 = ab^2c^3 + abcc$
$4 + a^2b^2c^2$	5	$a^2bcx + bcdx + ab^3c^2 = ab^2c^3 + abcc + a^2b^2c^2$
$5 - ab^3c^2$	6	$a^2bcx + bcdx = ab^2c^3 + abcc + a^2b^2c^2 - ab^3c^2$
$6 \div$	7	$x = \frac{ab^2c^3 + abcc + a^2b^2c^2 - ab^3c^2}{a^2bc + bcd} = \frac{79936}{7104} = 39.4$

### General Directions for solving Mathematical Problems.

**W**HEN a Question is proposed to be resolved algebraically, put Letters for all the Quantities concern'd therein, whether known or unknown, according to the algebraic Method of Notation; and suppose that the Thing required is done. Then proceed exactly as if the Answer was really known, and you were going to try whether it is true or not; and this is to be done by truly stating all the Conditions of the Question, which will furnish you with one or more Equations, that must be cleared by the Rules given before in the *Reduction of Equations*.

For if the Signs in Algebra be rightly understood, the very writing down the Conditions of the Problem will furnish you with a proper Number of Equations, by which how to know whether the Problem be limited or not. Observe the following Rules.

#### R U L E I.

When the Number of Quantities sought exceeds the Number of given Equations, not depending on one another, the Question is capable of innumerable Answers.

#### R U L E

## RULE II.

*When the Number of the Equations, not depending on one another, are just as many as the Number of the Quantities sought, then is the Equation truly limited.*

## SCHOLIUM I.

Problems are more than limited, when the Number of unknown Quantities is less than the Number of independent Equations, in which Case it often becomes impossible.

## SCHOLIUM II.

A Problem will be impossible, tho' the Number of Equations be less than the Number of unknown Quantities, if they involve any contradiction; as if  $x, y, z$ , be three unknown Quantities, and if  $a - c + 2y = b$ .

$$\text{and } 2a - 2c + 4y = 3b.$$

Now since, by the first Equation,  $2a - 2c + 4y = 2b$ , therefore  $2b$  would be  $= 3b$ , which is a Contradiction, if  $b$  be any real Quantity.



# The Solution of Equations.

## Of Simple or Pure Equations.

**T**HE Reader having now well acquainted himself with the preceding Rules, it may not be improper, if we come now to shew the Manner of applying the same in solving mathematical Problems; to do which we have thought it necessary here, to insert some of the easiest Problems we could think of, and if the Method of Solution, we have used, be well observed, our young Learners will find no Difficulty in resolving those that are more intricate.

### P R O B L E M I.

*A Man being 100 Years old, upon his Birth-Day had his three Sons with him at Dinner, namely, William, James and Thomas. The Father saying to them, Well, Sons, I am this Day just 100 Years old; William the youngest, said, Father, my Brother Thomas is four times as old as I am, and my Brother James three times as old as I am, and all our Ages together are just your Age. How old was each of the three Sons?*

### S O L U T I O N.

Put  $x =$  William's Age; then because Thomas is four times older than William, his Age will be  $4x$ ; and because James is three times as old as William, his Age will be denoted by  $3x$ ; now William says that all their Ages added together is just the Father's Age, or 100.

For  $1 \mid x =$  William's Age.

Then  $2 \mid 4x =$  Thomas's Age.

And  $3 \mid 3x =$  James's Age.

$1 + 2 + 3 \mid 4 \mid 8x = 100$  by the Question.

$4 \div 8 \mid 5 \mid x = \frac{100}{8} = 12 \text{ Years } 6 \text{ Months}$  William's Age.

Consequently  $4x = 50$  Years, Thomas's Age, and  $3x = 37$  Years 6 Months, James's Age.

P R O B-

## PROBLEM II.

**T**HERE were in Company together four Persons, Adam, Edward, Charles and William. Adam told Edward that he was older than him by two Years; Charles told them that he was as old as both of them together, and four Years over. William hearing them say so, said, I am just 96 Years old, and that is equal to all your Ages. How old were Adam, Edward, and Charles?

## SOLUTION.

Let *Edward's* Age be denoted by  $x$ ; then *Adam* being 2 Years older, his Age will be  $x+2$ ; and *Charles* being as old as both, and 4 Years over, his Age will be  $x+x+2+4$ , that is,  $2x+6$ . But *William* said all their ages added together was just equal to his.

For	+	{	1	$x$	= <i>Edward's</i>	} Age = { <i>William's</i>	
			2	$x+2$	= <i>Adam's</i>		
			3	$2x+6$	= <i>Charles's</i>		
			+	2	+	3	
			4	$4x+8$	= 96	Equation.	
			4	- 8			
			5	$4x$	= 96 - 8 = 88		
			5	$\div 4$			
			6	$x = \frac{88}{4} = 22$ ,	<i>Edward's</i> Age		
Consequently			7	$x+2 = 24$ ,	<i>Adam's</i> Age		
And			8	$2x+6 = 50$ ,	<i>Charles's</i> Age.		

## PROBLEM III.

**T**HREE Persons, Andrew, Benjamin and Charles, are to go a Journey of 340 Miles; of this Journey, Andrew is to go a certain Number of Miles unknown; Benjamin is to go four times as many Miles as Andrew, and three Miles more; and Charles is to go twice as many Miles as Benjamin, and five Miles more. How many Miles must each of these Persons travel severally, to make out their Journey 340 Miles?

## SOLUTION.

Put  $x$  = Number of Miles that *Andrew* went; then *Benjamin* will go  $4x+3$ , and *Charles* will go  $8x+6+5$ ,  
or



or  $8x + 11$ , because he goes twice as many miles, and five more than *Charles*; all which added together must be equal to 340, whence this

Equation  $1 \mid 13x + 14 = 340$

$1 - 14 \mid 2 \mid 13x = 340 - 14 = 326$

$2 \div 13 \mid 3 \mid x = \frac{326}{13} = 25\frac{1}{13}$ , the Miles *Andrew* went

Then  $4 \mid 103\frac{1}{13}$  the Miles that *Benjamin* went, and  $211\frac{5}{13}$  the Miles that *Charles* went.

For  $\begin{cases} 25\frac{1}{13} \\ 103\frac{1}{13} \\ 211\frac{5}{13} \end{cases}$

Proof  $\underline{340}$

PROBLEM IV.

**HERE** is 2731. to be divided amongst 4 Persons, namely, *Andrew*; *Bennet*; *Christopher* and *Daniel*. *Andrew* is to have a Share unknown; *Bennet* is to have twice as much as *Andrew*, and 301. more; *Christopher* is to have 3 times as much as *Andrew*, wanting 521. and *Daniel* is to have five times as much as *Andrew*, and 201. more. How must this 2731. be divided amongst them, that every one may have his true Share?

SOLUTION.

Let  $x$  denote *Andrew's* Share; then *Bennet's* will be  $2x + 30$ , *Christopher's* will be  $3x - 52$ , and *Daniel's*  $5x + 20$ , according to the Nature of the Question; which being added together, the Sum must be equal to 2731; whence this

Equation  $1 \mid 11x + 30 + 20 - 52 = 273$

that is  $2 \mid 11x - 2 = 273$

$2 + 2 \mid 3 \mid 11x = 273 + 2 = 275$

$3 \div 11 \mid 4 \mid x = \frac{275}{11} = 25$ , *Andrew's* Share

Whence *Bennet's* = 80, *Christopher's* = 23, and *Daniel's* = 145.

For  $\begin{cases} 25 \\ 80 \\ 23 \\ 145 \end{cases}$   
 $\underline{273}$

## PROBLEM V.

*A Man dying, gave to his eldest Son  $\frac{2}{3}$  of  $\frac{1}{4}$  of his Estate, to his second Son  $\frac{1}{3}$  of  $\frac{1}{2}$  of his Estate; and when they had counted their Portions, the eldest Son had 40*l.* more than the other: the remainder of the Estate was given to the Wife and younger Children. The Question is, how much was the first and second Son's Portion? and how much was left to the Wife and younger Children?*

## SOLUTION.

Let  $x$  be put for the whole Estate; then  $\frac{2}{3}$  of  $\frac{1}{4}$  of  $x$  is  $\frac{1}{6}x$ , or  $\frac{1}{6}x$  = the eldest Son's Share; and  $\frac{1}{3}$  of  $\frac{1}{2}$  of  $x$ , or  $\frac{1}{6}x$ , is the second Son's Share; but the Question says, the eldest Son had 40*l.* more than the second; therefore to bring it to an Equation, add 40*l.* to the second Son's Share, and it will be  $\frac{1}{6}x + 40$ ; which, by the Question, is equal to the eldest Son's Share; whence this

Equation	1   $\frac{1}{6}x = \frac{1}{6}x + 40$
1 — $\frac{1}{6}x$	2   $\frac{1}{6}x - \frac{1}{6}x = 40$
that is	3   $\frac{1}{15}x = 40$
3 $\times$ 15	4   $x = 600$ <i>l.</i> the whole Estate.

Then  $\frac{1}{6}x = 100$  *l.* the eldest Son's Share.

$\frac{1}{6}x = 60$  *l.* the second Son's Share.

And there was 440*l.* for the Wife and younger Children.

## PROBLEM VI.

*ONE coming into an Orchard asked the Gardiner how many Trees there were in the Orchard; the Gardiner answered, that the one half of the Trees were Apple-Trees, a fourth Part Pear-Trees, the seventh Part were Plum-Trees, and that there were 12 Cherry-Trees besides. How many Trees were there in the Orchard?*

## SOLUTION.

Put  $x$  = the Number of Trees that were in the Orchard; then  $\frac{x}{2}$  will be the Number of Apple-Trees,

$\frac{x}{4}$  the Pear-Trees,  $\frac{x}{7}$  the Plumb-Trees, and 12 Cherry-Trees; all which being added together must, by the Question, be equal to  $x$ , the total Number of Trees in the Orchard; which being done, we have this

$$\text{Equation } 1 \quad \frac{x}{2} + \frac{x}{4} + \frac{x}{7} + 12 = x$$

$$\text{Contrac. } 2 \quad \frac{50x}{56} + 12 = x, \text{ by the Rules of Fractions.}$$

$$3 \times 56 \quad 3 \quad 50x + 672 = 56x$$

$$4 \quad 40x \quad 4 \quad 56x - 50x = 672$$

$$\text{that is } 5 \quad 6x = 672$$

$$6 \div 6 \quad 6 \quad x = 112, \text{ the Number of Trees in the Orchard.}$$

Whence there were 56 Apple-Trees, 28 Pear-Trees, 16 Plumb-Trees; which added to the 12 Cherry-Trees make 112.

# PROBLEM VII.

**A.** begins the World with a certain Stock of Money; which he improv'd so well by Trade, that at the End of the first Year he doubled his Stock, except 100l. expended for the Use of his Family; and so he goes on every Year doubling his last Year's Stock, except 100l. a Year's Expenses, and at the End of three Years he found himself three times as rich as at first. I demand his first Stock?

# SOLUTION.

Put  $x$  = his first Stock; then his Stock at the End of the first Year is

$$2x - 100$$

Second Year's End

$$4x - 300$$

Third Year's End

$$8x - 700$$

Equation

$$8x - 700 = 3x$$

$$\text{Equation } 1 \quad 8x - 700 = 3x$$

$$1 + 700 \quad 2 \quad 8x = 3x + 700$$

$$2 - 3x \quad 3 \quad 8x - 3x = 700$$

$$\text{that is } 4 \quad 5x = 700$$

$$4 \div 5 \quad 5 \quad x = \frac{700}{5} = 140$$

*Proof.* For  $140 \times 8 = 1120$ , and  $1120 - 700 = 420$ ,  
and  $140 \times 3 = 420$ .

## P R O B L E M VIII.

**T**HERE is a Fish whose Head is 9 Inches long, and the Tail as long as the Head and half the Body, and the Body as long as the Head and Tail together. Quere, the length of the Body and Tail separately?

## S O L U T I O N.

The length of the Body put  $x$ .

Then the length of the Tail will be  $\frac{x}{2} + 9$ .

$$\text{Equation } x = \frac{x}{2} + 9 + 9.$$

$$\text{Equation } \left| \begin{array}{l} x = \frac{x}{2} + 18 \end{array} \right.$$

$$1 \times 2 \quad \left| \begin{array}{l} 2x = x + 36 \end{array} \right.$$

$2 - x \quad \left| \begin{array}{l} 3x = 36 \text{ the length of the Body,} \\ \text{and } 27 \text{ the length of the Tail.} \end{array} \right.$

## P R O B L E M IX.

**R**Equired to divide 100 into two such Parts, that  $\frac{3}{4}$  of one Part subtracted from  $\frac{1}{2}$  of the other may leave 39?

## S O L U T I O N.

For the first Part put  $x$

Then by Ax. 3. the other is  $100 - x$

$\frac{3}{4}$  of one Part  $\frac{3x}{4}$

And  $\frac{1}{2}$  of the other  $\frac{500 - 5x}{6}$

$$\text{Equation } \frac{500 - 5x}{6} - \frac{3x}{4} = 39.$$

Equation	1	$\frac{500-5x}{6} - \frac{3x}{4} = 39$
$1 \times 6$	2	$500-5x - \frac{18x}{4} = 234$
$2 \times 4$	3	$2000-20x-18x=936$
$3+20x$	4	$2000-18x=936+20x$
$4+18x$	5	$2000=936+20x+18x$
$5-936$	6	$2000-936=20x+18x$
that is	7	$38x=1064$
$7 \div 38$	8	$x = \frac{1064}{38} = 28.$

PROBLEM X.

**A** and B engage at Play, A had 100 Guineas, and B 80, before they begin; and, after a certain Number of Games won and lost between them, A rises with three times as many Guineas as B; I demand how many Guineas A won of B?

SOLUTION.

The Number A. won of B. put  $x$ .

A.'s Number at last  $x+100$ .

B.'s at last  $80-x$ .

Equation  $100+x=240-3x$ .

Equation	1	$100+x=240-3x$
$x+3x$	2	$100+4x=240$
$2-100$	3	$4x=140$
$3 \div 4$	4	$x = \frac{140}{4} = 35.$

PROBLEM XI.

**O**NE meeting a Company of Beggars, gave to each 6 Pence; and had 20 Pence over; but if he had given 8 Pence each, he would have wanted 16 Pence for that Purpose. Quere, the Number of Beggars?

SOLUTION.

The Number of Beggars put	$x$ .
The Number of Pence given	$6x$ .
The Number of Pence in all	$6x+20$ .

The Number of Pence that would have been }  $8x$   
 given at 8 Pence

Then he would have wanted 16 Pence, viz.  $8x - 16$

$$\text{Equation } 6x + 20 = 8x - 16.$$

$$\text{Equation } \begin{array}{l|l} 1 & 6x + 20 = 8x - 16. \\ 1 + 16 & 2 \end{array} \quad \begin{array}{l} 6x + 20 = 8x - 16. \\ 6x + 36 = 8x. \end{array}$$

$$2 - 6x \quad \begin{array}{l|l} 3 & 8x - 6x = 36. \\ 3 & 8x - 6x = 36. \end{array}$$

$$\text{that is } \begin{array}{l|l} 4 & 2x = 36. \\ 4 & 2x = 36. \end{array}$$

$$4 \div 2 \quad \begin{array}{l|l} 5 & x = \frac{36}{2} = 18. \\ 5 & x = \frac{36}{2} = 18. \end{array}$$

$$\text{For } 18 \times 6 = 108.$$

$$\text{And } 18 \times 8 = 144$$

$$+ 20$$

$$\frac{128}{128}$$

$$- 16$$

$$\frac{128}{128}$$

### PROBLEM XII.

**O**NE lets out a certain Sum of Money at 4 per Cent. Simple Interest, which, in 12 Years, wanted but 20l. of the Principal. What was the Principal?

### SOLUTION.

For the Principal put  $x$

The Interest for a Year, }

$$\text{For } 100 : 4 :: x : \frac{4x}{100} \quad \left. \vphantom{\begin{array}{l} 100 : 4 \\ x : \frac{4x}{100} \end{array}} \right\} \quad \frac{4x}{100}$$

$$\text{For 12 Years} \quad \frac{48x}{100} \text{ or } \frac{12}{25}x.$$

$$\text{Equation } \frac{12}{25}x = x - 20.$$

$$\text{Equation } \begin{array}{l|l} 1 & \frac{12}{25}x = x - 20. \\ 1 \times 25 & 2 \end{array} \quad \begin{array}{l} \frac{12}{25}x = x - 20. \\ 12x = 25x - 500. \end{array}$$

$$2 + 500 \quad \begin{array}{l|l} 3 & 500 + 12x = 25x. \\ 3 & 500 + 12x = 25x. \end{array}$$

$$3 = 12x \quad \begin{array}{l|l} 4 & 500 = 25x - 12x. \\ 4 & 500 = 25x - 12x. \end{array}$$

$$\text{hence } \begin{array}{l|l} 5 & x = \frac{500}{13} = 38\frac{6}{13}. \\ 5 & x = \frac{500}{13} = 38\frac{6}{13}. \end{array}$$

### PROBLEM XIII.

**R**Equired to divide 24s. into 24 Pieces, consisting only of Nine-pences, and Thirteen Pence Half-pennies?

S O L U -

## SOLUTION.

For the Number of Nine-pences put  $x$   
 By  $Ax$ .  $3d$ . of  $13d$ .  $\frac{1}{2}$   $24-x$   
 Of Half-pence = the former  $18x$   
 The latter  $648-27x$   
 The whole  $18x+648-27x$   
 Equation  $18x+648-27x=576$ .

Equation	1	$18x+648-27x=576$ .
	1	$+27x$
	2	$18x+648=576+27x$ .
	2	$-18x$
	3	$648=576+9x$ .
	3	$-576$
	4	$72=9x$ .
	4	$\div 9$
	5	$x=\frac{72}{9}=8$ Nine-pences, and $24-8=16, 13d$ .

To strengthen the Idea of the Learner, we have all along supposed  $x$  for the unknown Quantity, and the known ones by their respective Figures. But the same may be done more generally, and at the same time more elegantly, by substituting  $a, b, c, d$ , &c. for known Quantities, and  $x, y, z, v$ , &c. for unknown. Besides, it will be more conducive to *reasoning*, because it serves *universally*, in all Cases, and does not in the least appear as if *contrived*, and therefore is preferable to any other, to represent the known and unknown Quantities by Algebraic Symbols; since from thence a general Theorem is derived, whereby all other Questions of the same kind may be resolved.

## PROBLEM XIV.

**TWO** Men, A. and B. travelling together, A. with 240l. and B. with 96l. met a Company of Robbers, who took twice as much from A. as from B. and left A. thrice as much as they left B. How much did they take from each?

## SOLUTION.

Put

$240=a$

$96=b$

$2=c$

$3=d$

Sum taken from B =  $x$ 

P 3

Then

Then from A  $= cx$ By  $Ax. 3$   $\left\{ \begin{array}{l} \text{Left B} = b - x \\ \text{Left A} = a - cx \end{array} \right.$ Equation  $a - cx = db - dx$ .Equation  $1 \mid a - cx = db - dx$  $1 + dx \mid 2 \mid a - cx + dx = db$ . $2 - a \mid 3 \mid dx - cx = db - a$ . $3 \div \mid 4 \mid x = \frac{db - a}{a - c} = 48$ .

## PROBLEM XV.

SUPPOSE a Cask holds 81 Gallons of Wine when full, out of which a certain Quantity is exhausted, and then the Cask is filled up again with Water; the same Quantity being again drawn out as at first, and the Cask again filled up with Water, and so on four Times, always filling the Cask with Water after every Evacuation, there is at last found 16 Gallons of Wine left in the Cask besides Water. What Quantity of Wine was drawn out each Time?

## SOLUTION.

Since there was the same Quantity of Liquor in the Vessel before every Draught was made, viz. 81 Gallons, and since there was taken the same Quantity at every Draught, it follows that there must be left the same Quantity after every Draught. Call this remainder  $x$ , and then we may observe, that at every Draught the whole Quantity of Liquor in the Vessel, and consequently the Wine, will be diminish'd in the Proportion of 81 to  $x$ , and tho' the Water was afterwards recruited, the Wine was not; therefore if  $x$  represents the Wine left in the Vessel after the first Draught, and  $a$  be put  $= 81$ ,  $b = 16$ , the Wine after the second Draught may be found by saying

As  $a : x :: x : \frac{x^2}{a} =$  the Wine left after the second Draught.

Also  $a : x :: \frac{x^2}{a} : \frac{x^3}{a^2} =$  Wine left after the 3d Draught.

And



And  $a : x :: \frac{x^3}{a^2} : \frac{x^4}{a^3} =$  Wife left after the fourth

Draught, which by the Question must be equal to 16 Gallons. Hence this

Equation  $\frac{x^4}{a^3} = b.$

Equation	1	$\frac{x^4}{a^3} = b.$
$1 \times a^3$	2	$x^4 = a^3 b$
$2 u u 2$	3	$x^2 = \sqrt{a^3 b}$
$3 u u 2$	4	$x = \sqrt[4]{a^3 b} = 54.$

For  $\frac{x^4}{531441} = 16$

$x^4 =$	8503056
$x^3 =$	157464
$x^2 =$	2916
$x =$	54

P R O O F.

81—54=27, first Draught.  
 54—36=18, second Draught.  
 36—24=12, third Draught.  
 24—16= 8, fourth Draught.  
 65+16=81. Q. E. I.

P R O B L E M XVI.

*A* Clock has two Hands turning upon the same Center, whereof the swifter makes a Revolution every 12 Hours, and the slower every 16. Quere, the Synodical Period of the two Hands ?

*N. B.* That by a synodical Period, I mean the whole Time from the Moment the two Hands are together to the Moment they come together again; whence it follows that, in every synodical Period, the swifter Hand makes one Revolution more than the slower. For if we once suppose the two Hands together, it will be impossible for them to come together again, till the swifter Hand has got an entire Revolution before the slower, in which time it must have made one Revolution more than the slower; whence the Solution is as follows:

## SOLUTION.

Put  $12=a$ ,  $16=b$ , difference  $=d=4$ ,  
 Number of Hours in a synodical Period  $=x$ .

Number of Revolutions in the swifter  $=\frac{x}{a}$ .

In the Slower  $=\frac{x}{b}$ .

The Difference  $=\frac{x}{d}$ .

$$\text{Equation } \frac{x}{a} = \frac{x}{b} + 1.$$

$$\text{Equation } 1 \frac{x}{a} = \frac{x}{b} + 1.$$

$$1 \times a \quad 2x = \frac{ax}{b} + a.$$

$$2 \times b \quad 3bx = ax + ab.$$

$$3 - ax \quad 4bx - ax = ab.$$

$$4 \div b - a \quad 5x = \frac{ab}{b-a} = 48.$$

## PROBLEM XVII.

ONE buys a certain Number of Eggs, half at 2 a Penny, and the other half at 3 a Penny. Afterwards selling out the Whole at 5 for Two-pence, he contrary to his Expectation lost 12d. by his Bargain. How many were bought in at first?

## SOLUTION.

Put  $12=a$ ,  $2=b$ ,  $3=c$ ,  $5=m$ .

Number bought at first  $x$

Number at 2 a Penny  $\frac{x}{b}$

Number at 3 a Penny  $\frac{x}{c}$

Price of the former in Pence  $\frac{x}{4}$

Of the latter  $\frac{x}{6}$

Of

Of the Whole

$$\frac{10x}{ab}$$

Equation  $\frac{10x}{ab} = \frac{bx}{m} + a$ .

Equation	1	$\frac{10x}{ab} = \frac{bx}{m} + a$ .
$1 \times ab$	2	$10x = \frac{abbx}{m} + aab$ .
$2 \times m$	3	$10mx = abbx + aabm$ .
$3 - ab^2x$	4	$10mx - abbx = aabm$ .
$4 \div 10m - abb$	5	$x = \frac{aabm}{10m - abbx} = \frac{1440}{2} = 720$ .

PROBLEM XVIII.

**FOUR** Gamesters A, B, C, D, each with a different Stock of Money about him, sit down to play; A. wins  $\frac{1}{2}$  of B.'s first Stock, and at the same Time B. wins  $\frac{1}{3}$  of C.'s, and C.  $\frac{1}{4}$  of D.'s, and D.  $\frac{1}{5}$  of A.'s; after which they each rise with 23 Guineas. How many Guineas had each at first?

SOLUTION.

The Money D. won of A. put	$x$
A.'s first Stock	$5x$
The Remainder of A.'s first Stock after his loss to D.	$4x$
The Money A. won of B.	$23 - 4x$
B.'s first Stock	$46 - 8x$
The Remainder of B.'s first Stock after his Loss to A.	$23 - 4x$
The Money B. won of C.	$= 4x$
C.'s first Stock	$12x$
Remainder of C.'s first Stock after his Loss to B.	$8x$
The Money C. won of D.	$23 - 8x$
D.'s first Stock	$92 - 32x$
The Remainder of D.'s first Stock after his Loss to C.	$69 - 24x$
Another Expression. D. won of A.	$= 24x - 46$
Equation	$24x - 46 = x$ .

Equation

$$\text{Equation} \begin{array}{l|l} 1 & 24x - 46 = x. \\ 2 & 24x = x + 46. \\ 3 & 23x = 46. \\ 4 & x = \frac{46}{23} = 2, \text{ the Money } D. \text{ won of } A. \end{array}$$

$$1 + 46$$

$$2 - x$$

$$3 \div 23$$

$$4 | x = \frac{46}{23} = 2, \text{ the Money } D. \text{ won of } A.$$

## PROBLEM XIX.

*Required to divide 180 into two such Parts that one may be to the other as 2 to 3?*

## SOLUTION.

Put  $2 = a$ ,  $3 = b$ ,  $180 = m$ . Let  $x =$  one Part, and by Ax. 3.  $m - x$  will be the other Part. Proportion  $x : m - x :: a : b$ . Therefore  $bx = am - ax$ . By Prop. IX.

$$\text{Equation} \begin{array}{l|l} 1 & bx = am - ax. \\ 2 & bx + ax = am. \\ 3 & x = \frac{am}{b + a} = 72. \end{array}$$

$$1 - ax$$

$$2 \div b + a$$

## PROBLEM XX.

*THERE is a Pole 6 Feet long, at whose Extremities are suspended 2 Weights, one of 5 Pounds, the other of 7; I demand the common central Gravity of the Weights, that is, its Distance from the two Extremities of the Pole?*

## SOLUTION.

$$\text{Let } 6 \text{ feet} = 72 \text{ inches} = a.$$

$$5 = b, \quad 7 = c.$$

Dist. of its central Gravity from 7 Pound weight  $x$

From the 5 by Ax. 3.

Proportion

$$x : a - x :: b : c$$

Therefore  $cx = ab - bx$ . By Prop. IX.

$$\text{Equation} \begin{array}{l|l} 1 & cx = ab - bx. \\ 2 & cx + bx = ab. \\ 3 & x = \frac{ab}{c + b} = 30. \end{array}$$

$$1 + bx$$

$$2 \div c + b$$

## PROBLEM XXI.

**W**HAT Number is that, which being severally added to 36 and 52, will leave the former Sum to the latter as 3 to 4?

## SOLUTION.

Put  $36 = a$ ,  $52 = b$ ,  $3 = c$ ,  $4 = d$ .

Number sought  $= x$ .

Proportion  $a + x : b + x :: c : d$ .

Th.  $ad + dx = bc + cx$ . By Prop. IX.

Equation 1  $ad + dx = bc + cx$ .

1  $- cx$  2  $ad + dx - cx = bc$ .

2  $- ad$  3  $dx - cx = bc - ad$ .

3  $\div d - c$  4  $x = \frac{bc - ad}{d - c} = 12$ .





## Of Transferring and exterminating the unknown Quantities.

### R U L E.

**FIND** by Reduction the Value of one unknown Quantity in one Equation, and substitute this Value for it in the other Equations.

### E X A M P L E.

Let  $a+x=2b-y$ , and  $3ax-yx=d$ ; to exterminate  $y$ . By the first Equation I so transpose the Quantities  $a$  and  $x$ , till I get  $y=2b-a-x$ ; now seeing  $2b-a-x$  is the same as  $y$ , substitute this Value of  $y$  in the second Equation, and it gives  $3ax-xx \times 2b-a-x=d$ , that is,  $3ax-2bx-ax+xx=d$ ; but because  $3ax-ax=2ax$ , it will be  $2ax-2bx+xx=d$ .

We have purposely omitted other Examples that fall under this Denomination, because this would swell the Book too big.

### P R O B L E M XXII.

**THERE** is a certain Fraction, which, if an Unit be added to the Numerator, will be equal to  $\frac{1}{3}$ ; but if, on the contrary, an Unit be added to the Denominator, the Fraction then will be equal to  $\frac{1}{4}$ . Quere, the Numerator and Denominator?

### S O L U T I O N.

For the Fraction sought put  $\frac{x}{y}$ .

Then  $\left\{ \begin{array}{l} 1 \left| \frac{x+1}{y} = \frac{1}{3} \right. \\ 2 \left| \frac{x}{y+1} = \frac{1}{4} \right. \end{array} \right\} \text{ by the Question.}$

1x y	3	$x + 1 = \frac{y}{3}$ .
3x 3	4	$3x + 3 = y$ .
2xy + 1	5	$x = \frac{y + 1}{4}$ .
5x 4	6	$4x = y + 1$ .
4 - 3	7	$3x = y - 3$ .
7 ÷ 3	8	$x = \frac{y - 3}{3}$ .
6 ÷ 4	9	$x = \frac{y + 1}{4}$ .
8 = 9	10	$\frac{y - 3}{3} = \frac{y + 1}{4}$ , here $x$ is exterminated.
10x 3	11	$y - 3 = \frac{3y + 3}{4}$ .
11x 4	12	$4y - 12 = 3y + 3$ .
12 + 12	13	$4y = 3y + 3 + 12 = 3y + 15$ .
13 - 3y	14	$4y - 3y = 15$ .
that is	15	$y = 15$ .

From the 9th Step  $x = 4$ ; so  $\frac{4}{3}$  is the Fraction.

### PROBLEM XXIII.

**T**HERE is a certain Bowling-green, which, if it was 2 foot broader, and 3 longer, would be 64 square Feet; but if, on the other hand, it was 3 foot broader, and 2 longer, it would be 68 square Feet. Quere the Dimensions of the Bowling-green?

### SOLUTION.

Let  $64 = a$ ,  $68 = b$ ,  $3 = c$ ,  $2 = d$ , and let  $x$  and  $y$  denote the Breadth and Length respectively; then its area will be  $xy$ , the Breadth and Length of it on the first Supposition will be  $x + d$ , and  $y + c$ ; now the Length multiplied by the Breadth gives the Area, that is,  $x + dx$   $y + c = xy + dy + cx + cd$ . The Breadth and Length on the second Supposition will be  $x + c$  and  $y + d$ , and for the same Reasons above its Area will be  $xy + cy + dx + cd$ . Whence we have these

Q

Equa-

Equations	{	1	$xy + dy + cx + cd = xy + a.$
		2	$xy + cy + dx + cd = xy + b.$
1 = xy		3	$dy + cx + cd = a.$
3 = dy		4	$cx + cd = a - dy.$
4 = cd		5	$cx = a - dy - cd.$
5 ÷ c		6	$x = \frac{a - cd - dy}{c}.$
2 = xy		7	$cy + dx + cd = b.$
7 = cy		8	$dx + cd = b - cy.$
8 = cd		9	$dx = b - cy - cd.$
9 ÷ d		10	$x = \frac{b - cd - cy}{d}.$
6 = 10		11	$\frac{a - cd - dy}{c} = \frac{b - cd - cy}{d},$ here $x$ vanishes.
11 x c		12	$a - cd - dy = \frac{bc - ccd - ccy}{d}.$
12 x d		13	$ad - cdd - ddy = bc - ccd - ccy.$
13 ±		14	$ccy - ddy = bc - c^2d + cd^2 - ad.$
14 ÷		15	$y = \frac{bc - c^2d + cd^2 - ad}{c^2 - d^2} = \frac{13}{5} = 14.$

Consequently  $x = 10.$

*Note,*  $\pm$  denotes transposition, and signifies that the 13 step is so order'd as to get the unknown Quantities on one side of the Equation, and the known ones on the other; and is no more but saving the Trouble of setting down a Step or two.

### PROBLEM XXIV.

ONE lays out 2s. 6d. in Apples and Pears, buying his Apples at 4 a Penny, and his Pears at 5; and afterwards accommodates his Neighbour with  $\frac{1}{2}$  of his Apples, and  $\frac{1}{3}$  of his Pears for 13d. the Price he bought them at. Quere how many were bought of each Sort?

### SOLUTION.

Let 2s. 6d. = 30d. =  $a$ , 4 =  $b$ , 5 =  $c$ , 13 =  $d$ , Number of Apples =  $x$ , Number of Pears =  $y$ .

Price



Price of the former in Pence (by *Ax. 4.*)

$$\frac{x}{b}$$

Price of the latter in Pence

$$\frac{y}{c}$$

Price of half the former

$$\frac{x}{2b}$$

Price of one third of the latter

$$\frac{y}{3c}$$

Equations  $\begin{cases} \frac{x}{b} + \frac{y}{c} = a, & \text{By Reduction,} & x = \frac{acb - by}{c} \\ \frac{x}{2b} + \frac{y}{3c} = d, & \text{And} & x = \frac{6bcd - 2by}{3c} \end{cases}$

Whence  $x$  is expunged, and we have this

Equation 1  $\frac{acb - by}{c} = \frac{6bcd - 2by}{3c}$

1  $\times c$  2  $acb - by = \frac{6bcd - 2by}{3c}$

2  $\times 3c$  3  $3acb - 3by = 6bcd - 2by$

3  $\div$  4  $by = 3ac^2b - 6bc^2d$

4  $\div bc$  5  $y = \frac{3ac^2b - 6bc^2d}{bc} = \frac{1200}{20} = 60$

Hence he bought 60 Pears, and 72 Apples.

## PROBLEM XXV.

**T**HERE is a Number, consisting of two Places, which is equal to 4 Times the Sum of its Digits; and if to the Number be added 18, the Digits will be inverted. Quere the Number?

## SOLUTION.

The Digit in the Place of Tens

$$x. \quad 4 = a.$$

In the Place of Units

$$y. \quad 18 = b.$$

Sum of the Digits

$$x + y.$$

The Number represented by the Digits

$$\left\{ \begin{array}{l} 10x + y \end{array} \right.$$

The Number represented by the Digits inverted

$$\left\{ \begin{array}{l} 10y + x. \end{array} \right.$$

Q 2.

Equ

Equations	}	1	$10x + y = ax + ay.$
		2	$10y + x = 10x + y + b.$
$1 - y$		3	$10x = ax + ay - y.$
$3 - ax$		4	$10x - ax = ay - y.$
that is		5	$6x = 3y.$
$5 \div 6$		6	$x = \frac{3y}{6} = \frac{y}{2}.$
$2 \pm$		7	$x = \frac{9y - 18}{9}.$
$6 = 7$		8	$\frac{y}{2} = \frac{9y - 18}{9} = y - 2,$ hence $x$ vanishes.
$8 \times 2$		9	$y = \frac{18y - 36}{9} = 2y - 4.$
$9 + 4$		10	$y + 4 = 2y.$
$10 - y$		11	$y = 4.$

Hence 24 the Number sought  
42 the Digits inverted

### LEMMA.

In all Problems of three unknown Quantities, we must find three several Values of the first; then comparing these together, and resolving them, find two Values of the second; lastly, by comparing these two Values, find the value of the third.

So if there were four unknown Quantities, we must have four Equations of the first, three of the second, two of the third, and one of the last, or fourth, all found in the same Manner as above directed.

### PROBLEM XXVI.

**T**HREE Persons, A. B. and C. were talking of their Money; says A. to B. and C. give me  $\frac{1}{2}$  of your Money, and I shall have 17 Guineas; says B. to A. and C. give me  $\frac{1}{3}$  of your Money, and I shall have 17 Guineas; says C. to A. and B. give me  $\frac{1}{4}$  of your Money, and I shall have 17 Guineas. Quere the Number of Guineas each Man had?

SOLU-

SOLUTION.

For the Number of *A*'s Guineas put *x*.

Of *B*'s *y*.

Of *C*'s *z*.

$$\begin{array}{l} \text{Equations} \left\{ \begin{array}{l} x + \frac{y+z}{2} = 17. \\ y + \frac{x+z}{3} = 17. \\ z + \frac{x+y}{4} = 17. \end{array} \right. \end{array}$$

$$\begin{array}{l} \text{1st Equation} \left| \begin{array}{l} x + \frac{y+z}{2} = 17. \\ \text{Reduced} \quad 2x = \frac{34 - y - z}{2}. \end{array} \right. \end{array}$$

$$\begin{array}{l} \text{2d Equation} \left| \begin{array}{l} y + \frac{x+z}{3} = 17. \\ \text{Reduced} \quad 2x = 51 - 3y - z. \end{array} \right. \end{array}$$

$$\begin{array}{l} \text{3d Equation} \left| \begin{array}{l} z + \frac{x+y}{4} = 17. \\ \text{Reduced} \quad 2x = 68 - y - 4z. \end{array} \right. \end{array}$$

Now from the above three Equations we have,

First,  $\frac{34 - y - z}{2} = 51 - 3y - z$ , here *x* vanishes.

Reduced,  $y = \frac{68 - z}{5}$ .

Secondly,  $51 - 3y - z = 68 - y - 4z$ .

Reduced,  $y = \frac{3z - 17}{2}$ .

Lastly,  $\frac{68 - z}{5} = \frac{3z - 17}{2}$ ; here *y* vanishes.

Reduced,  $z = \frac{221}{17} = 13$  Guineas, *C*'s Number.

Whence  $y = 11$  Guineas, *B*'s Number.

And  $x = 5$  Guineas, *A*'s Number.



## Of the Resolution of adfected Quadratic Equations.

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**A**N adfected Quadratic Equation is an Equation including three different Sorts of Quantities; one Sort wherein the Square of the unknown Quantity is concerned; another Sort wherein it is simply concerned; and a third Sort wherein it is not concerned at all; as if  $24x - 2xx = xx + 45$ . For a Resolution whereof, we are first to say something of a *Binomial*: Now a Binomial is a Quantity consisting of two Members, connected together by the signs  $+$  or  $-$ ; as  $x + a$ , or  $a - x$ , or  $x + \frac{a}{2}$ , or  $x - \frac{a}{2}$ , &c. And a Square raised from a Binomial Root, is nothing else but the Square of such a Quantity.

Thus the Square of  $x + \frac{a}{2}$  is  $x + \frac{a}{2} \Big| ^2 = xx + ax + \frac{aa}{4}$ , and that of  $x - \frac{a}{2}$  is  $x - \frac{a}{2} \Big| ^2 = xx - ax + \frac{aa}{4}$ ; whence we may observe,

First, That whenever we meet with any Quantity consisting of two Members, as  $xx + ax$ , or  $xx - ax$ , whereof one, as  $xx$ , is a Square, and the other  $+ax$  is the Root of the Square multiplied into some given Coefficient  $+a$ ; whenever, I say, we meet with such a Quantity, it may be considered as the two first Members of an imperfect Square, raised from a Binomial Root, and may be easily compleated by adding  $\frac{aa}{4}$ , that is, by adding the Square of half the Coefficient of the Second. Thus,  $xx + 6x$ , when compleated, becomes  $xx + 6x + 9$ ; and  $xx - 8x$ , when compleated, becomes  $xx - 8x + 16$ .

Secondly, it may be observed that the Root of such a Square, thus compleated, will always be the Root of the first

first Member, together with half the Coefficient of the Second— Thus the Square-Root of  $xx+6x+9$  is  $x+3$ , and that of  $xx-8x+16$  is  $x-4$ . And this premis'd, the Resolution of Quadratic Equations is as follows.

*By completing the Square.*

R. U L E.

*ADD the Square of half the Coefficient of the unknown Quantity, to each side of the Equation, and the Square will be complet.*

E X A M P L E S.

1. $x^2+10x$	} When completed	$x^2+10x+25$	} Their Root	$x+5$
2. $x^2+12x$		$x^2+12x+36$		$x+6$
3. $x^2+3x$		$x^2+12x+\frac{9}{4}$		$x+\frac{3}{2}$
4. $x^2+5x$		$x^2+5x+\frac{25}{4}$		$x+\frac{5}{2}$

In resolving a Quadratic Equation, you must first of all clear it from Fractions; by the Rule already laid down, and by that means bring it into Integers; having so done, transpose the Terms, so that the two Powers of the unknown Quantity may solely possess one Side of the Equation, viz. that Side which will exhibit the Square or highest Power of the unknown Quantity. This Rule observed in the following Equation, we shall have  $3xx-24x=-45$ . In the next Place, since the highest Power, or  $xx$ , has a Coefficient before it, as 3, free it from the Coefficient, by dividing the whole Equation by 3, and we shall have  $xx-8x=-15$ ; and  $xx-8x$  must be considered as the two first Members of an imperfect Square, rais'd from a Binomial Root, which being compleated, according to the Rule above, we shall have  $xx-8x+16=1$ ; and, consequently, the Square Root of one Side being equal to the Square Root of the other, we have  $x-4=\pm 1$ ; I say,  $\pm 1$ , because either  $+1$ , or  $-1$ , multiplied into itself, produces  $+1$ ; but you transpose 4, to make it affirmative, and then  $x=4\pm 1$ ; so that in the Equation  $x=3$  or 5, each of which will equally satisfy the Conditions of the Problem: Whence it may be

be observed, that every Quadratic Equation admits of two Roots, or Numbers, that will answer it; and consequently all Problems, producing Quadratic Equations, do necessarily admit of two Solutions, which in Geometrical Problems, and in many others, are equally significant, tho' in Arithmetical Questions they are not equally useful, I mean when the Root of the Equation is negative.

N. B.  $c$ ,  $\square$ , set in the Margin of the Register Steps, denote the Square completed.

There are three Cases that fall under the Denomination of Quadratics with regard to finding the true Root or Value of the unknown Quantity, and they are known by the Signs  $+$  and  $-$ .

## C A S E I.

If  $xx + ax = b$ . Then  $x = \sqrt{b + \frac{1}{4}a^2} - \frac{1}{2}a$ .

For	1	$xx + ax = b$ .
1 $\square$	2	$xx + ax + \frac{1}{4}a^2 = b + \frac{1}{4}a^2$ .
2 $\cup \cup$ 2	3	$x + \frac{1}{2}a = \sqrt{b + \frac{1}{4}a^2}$
3 $-\frac{1}{2}x$	4	$x = \sqrt{b + \frac{1}{4}a^2} - \frac{1}{2}a$ .

## C A S E II.

If  $xx - ax = b$ . Then  $x = \sqrt{b + \frac{1}{4}a^2} + \frac{1}{2}a$ .

For	1	$xx - ax = b$ .
1 $\square$	2	$xx - ax + \frac{1}{4}a^2 = b + \frac{1}{4}a^2$ .
2 $\cup \cup$ 2	3	$x - \frac{1}{2}a = \sqrt{b + \frac{1}{4}a^2}$ .
3 $+\frac{1}{2}a$	4	$x = \sqrt{b + \frac{1}{4}a^2} + \frac{1}{2}a$ , or $= \frac{1}{2}a + \sqrt{b + \frac{1}{4}a^2}$ .

## C A S E III.

If  $ax - xx = b$ , or  $xx - ax + b = 0$ , or  $xx - ax = -b$ .  
Then  $x = \frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 - b}$

For	1	$xx - ax = -b$ .
1 $\square$	2	$xx - ax + \frac{1}{4}a^2 = \frac{1}{4}a^2 - b$ .
2 $\cup \cup$ 2	3	$x - \frac{1}{2}a = \sqrt{\frac{1}{4}a^2 - b}$ .
3 $+\frac{1}{2}a$	4	$x = \frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 - b}$ .

*Quadratics of an higher Nature.*

If  $x^4 + ax^2 = b^2$ . Then  $x = \sqrt{\frac{b^2 + \frac{1}{4}a^2}{2a}}$ .

For	1	$x^4 + ax^2 = b^2$ .
1 c □	2	$x^4 + ax^2 + \frac{1}{4}a^2 = b^2 + \frac{1}{4}a^2$ .
3 u w 2	3	$x^2 + \frac{1}{2}a = \sqrt{b^2 + \frac{1}{4}a^2}$ .
3 — $\frac{1}{2}a$	4	$x^2 = \sqrt{b^2 + \frac{1}{4}a^2} - \frac{1}{2}a$ .
4 u w 2	5	$x = \sqrt{\frac{b^2 + \frac{1}{4}a^2}{2a}}$ .

If  $x^6 + ax^3 = b^2$ . Then  $x = \sqrt[3]{\frac{b^2 + \frac{1}{4}a^2}{2a}}$ .

For	1	$x^6 + ax^3 = b^2$ .
1 c □	2	$x^6 + ax^3 + \frac{1}{4}a^2 = b^2 + \frac{1}{4}a^2$ .
2 u w 2	3	$x^3 + \frac{1}{2}a = \sqrt{b^2 + \frac{1}{4}a^2}$ .
3 — $\frac{1}{2}a$	4	$x^3 = \sqrt{b^2 + \frac{1}{4}a^2} - \frac{1}{2}a$ .
4 u w 3	5	$x = \sqrt[3]{\frac{b^2 + \frac{1}{4}a^2}{2a}}$ , or $\sqrt[3]{b^2 + \frac{1}{4}a^2 - \frac{1}{2}a}$ .

PROBLEM XXVII.

**W**HAT two Numbers are those, whose Sum is 27, and the Sum of their Squares 449?

SOLUTION.

Let  $27 = a$ ;  $449 = b$ .

Let one Number be

Then *per Ax. 3.* the other is

The Square of the former

Of the latter

Sum of their Squares

$x$   
 $a - x$

$xx$

$aa - 2ax + xx$

$aa - 2ax + 2xx$

Equation 1  $aa - 2ax + 2x^2 = b$ .

1 —  $a^2$  2  $2x^2 - 2ax = b - a^2$ .

2 ÷ 2 3  $x^2 - ax = \frac{b - aa}{2}$ .

3 c □ 4  $x^2 - ax + \frac{1}{4}aa = \frac{1}{4}aa + \frac{b - aa}{2}$ .

4 u w 2 5  $x - \frac{1}{2}a = \sqrt{\frac{1}{4}aa + \frac{b - aa}{2}}$ .

5 +  $\frac{1}{2}a$  6  $x = \frac{1}{2}a + \sqrt{\frac{1}{4}aa + \frac{b - aa}{2}} = \frac{27 \pm 13}{2} = 20 \text{ or } 7$ .

## PROBLEM XXVIII.

*A Man buys a Horse, and sells him again for 24*l.* and gain'd as much per Cent, as the Horse cost him; what, cost the Horse?*

*N. B.* Gain'd as much *per Cent.* is to be understood thus; he gain'd so much, that 100*l.* at the same rate of Gain would have gain'd the Money he laid out.

## SOLUTION.

For the Money laid out put  $x$ .

Then by *Ax.* 3. the gain  $24 - x$ .

Proportion  $x : 24 - x :: 100 : \frac{2400 - 100x}{x}$  = the

Gain *per Cent.*

$$\text{Equation } 1 \quad \frac{2400 - 100x}{x} = x.$$

$$1 \times x \quad 2 \quad 2400 - 100x = xx.$$

$$2 + 100x \quad 3 \quad xx + 100x = 2400.$$

$$3 \square \quad 4 \quad xx + 100x + 2500 = 4900.$$

$$4 \cup \cup 2 \quad 5 \quad x + 50 = \sqrt{4900} = 70.$$

$$5 - 50 \quad 6 \quad x = 70 - 50 = 20; \text{ the Horse cost him.}$$

He laid out 20*l.* and gain'd 4*l.* and 100*l.* at that Rate of Interest would have gain'd 20*l.* which is the Money he laid out.

For  $20 : 4 :: 100 : 20$ .

## PROBLEM XXIX.

*WHAT two Numbers are those, whereof twice the first, with three times the second, make 60, and twice the Square of the first, with 3 Times the Square of the second, make 840?*

## SOLUTION.

Let  $x$  and  $y$  denote the two Numbers sought.

Then  $2x + 3y = 60$ ; Thetef.  $x = \frac{60 - 3y}{2}$ .



Also  $2x^2 + 3y^2 = 840$ ; Theref.  $x^2 = \frac{840 - 3yy}{2}$ .

Equation 1  $\left| \frac{60 - 3y}{2} \right|^2 = \frac{840 - 3yy}{2}$ .

that is 2  $\frac{3600 - 360y + 9yy}{4} = \frac{840 - 3yy}{2}$ .

2x &c. 3  $7200 - 720y + 18yy = 3360 - 12yy$ .

3  $\pm$  4  $30yy - 720y = -3840$ .

4  $\div 30$  5  $yy - 24y = -128$ .

5  $\pm \square$  6  $y^2 - 24y + 144 = 144 - 128 = 16$ .

6  $\sqrt{\phantom{x}}$  7  $y - 12 = \sqrt{16} = 4$ .

7  $\pm 12$  8  $y = 12 \pm 4 = 16 \text{ or } 8$ .

And 9  $x = 18 \text{ or } 6$ .



## To find any Term in Arithmetical Progression.

---

**I**N Arithmetical Progression, any Term required may be made (without making all the antecedent Terms) by adding the first Term to the Product of the Difference (that is, the Number whereby all the Terms exceed one another) multiplied into the Distance of the Term required from the first. For Instance, in this Arithmetical Progression, 2, 4, 6, &c. where the first Term is 2, and also the Difference of each Term is 2, I would know what will be the twentieth Term, whose Distance consequently from the first Term is 19; wherefore  $19 \times 2 = 38$ , and  $38 + 2 = 40$ , which will constitute the 20th Term required in the Progression.

### L E M M A.

The Sum of any Series in Arithmetical Progression may be obtained by adding the greatest Terms together, and multiplying half that Sum into the Number of Terms, or the whole Sum into half the Number of Terms.

### P R O B L E M XXX.

*Suppose One sets out from London, and travels one Mile the first Day, 2 the second, 3 the third, &c. And 5 Days after another sets out from the same Place, and travels at the Rate of 12 Miles a Day. How long, and how far must the first have travelled before he was overtaken by the second?*

### S O L U T I O N.

For the Days travelled by the first put  $x$ .  
by the second  $5 - x$ .

Miles travelled by the first  $xx + \frac{x}{2}$ .

by the second  $12x - 60$ .

But before this Problem be brought to an Equation, it may not be improper to premise the following Observations.

I. You

1. You must observe that  $x$ , the Days travelled by the first, is equal both to the Number of Terms in the Progression, and also to the last Term. Then, *per Lemma*, the Sum of the whole Series is  $\frac{x^2+x}{2}$ . For 1

is the first Term, and  $x$  the last, and their Sum is  $x+1$ , and  $x+1 \times \frac{x}{2} = \frac{xx+x}{2}$ , the Sum of the whole Series or Number of Miles the first travelled.

2. If the last had travelled the same Number of Days as the first, he had travelled  $12x$  Miles; but as he set out 5 Days after, he must travel 5 times 60 Miles less than if he had travelled as many Days as the first; so he travelled  $12x-60$ . Then considering the Problem, we have this

Equation	1	$\frac{xx+x}{2} = 12x-60.$
reduc'd, &c.	2	$xx-23x = -120.$
2c□	3	$x^2-23x+\frac{529}{4} = -120+\frac{529}{4}.$
that is	4	$x^2-23x+\frac{529}{4} = \frac{-480+529}{4} = \frac{49}{4}.$
4w2	5	$x-\frac{23}{2} = \pm\frac{7}{2}.$
5+ $\frac{23}{2}$	6	$x = \frac{23 \pm 7}{2} = 15, \text{ or } 8.$

PROBLEM XXXI.

OUT of a common Pack of Cards (*viz.* 52.) let part be distributed into several distinct Heaps, in the manner following: Upon the lowest Card of every Heap, let as many others be laid as are sufficient to make up the Number 12; as if 4 be the Number of the bottom Card, let 8 others be laid upon it; if 5, let 7; if  $a$ , let  $12-a$ , &c. 'Tis required after having given the Number of Heaps, which we call  $n$ , as also the Number of Cards left in the Dealer's Hand, which we shall call  $m$ , to find the Sum of the Numbers of all the bottom Cards?

## SOLUTION.

Let  $a, b, c, d, e, f$ , &c. represent the bottom Cards of the several Heaps; then will  $12-a$  be the Number of Cards lying upon  $a$ , the first bottom Card; therefore  $13-a$  will be the Number of Cards in the first heap. In like manner  $13-b$  will be the Number of Cards in the second Heap,  $13-c$  in the third Heap, &c. Therefore if  $n$  be the Number of heaps,  $13n-a-b-c-d-e-f$ , &c. will be the Number of Cards in all the Heaps; and if  $x$  represents the Sum of all the bottom Cards,  $a+b+c+d+e+f$ , &c. then  $13n-x$  will be the Number of Cards contained in all the Heaps. But the Cards contained in all the Heaps will be the whole Pack, except the Number still remaining in the Dealer's Hands; therefore  $52-m$  will also represent all the Cards dealt out into the several Heaps, whence we have this

$$\begin{array}{l|l} \text{Equation} & 1 \\ \hline 1 \pm & 13n-x=52-m. \\ & 2 \quad x=13n-52+m. \end{array}$$

But  $13n$  is  $=n \times 13$ , and  $52=4 \times 13$ ; Th.  $13n-52=n-4 \times m$ ; Theref.  $x=n-4 \times 13-m$ , whence we have the following

## THEOREM.

*Subtract four from the Number of Heaps, multiply the rest by 13, and to the Product add (m) the Number of Cards remaining in the Dealer's Hands, and you'll have (x) the Number of all the bottom Cards.*

## EXAMPLE.

Suppose the Number of Heaps 3, and the Number of Cards left in the Dealer's Hands 30; what was the Sum of the Number of all the bottom Cards?

Answer 17. For  $13 \times 3 - 1 = 38$ , and  $38 - 30 = 8$ .

## PROBLEM XXXII.

YOU that are skill'd in mathematic Arts,  
Divide ten thousand into two such Parts,

When

When each of them the other hath divided,  
Both Quotients make just 5 when right decided?

SOLUTION.

Put  $a=10,000$ ,  $b=5$ ,  $x=$  one Part; by  $Ax. 3$ .  
 $a-x=$  other Part.

Equation	1	$\frac{x}{a-x} + \frac{a-x}{x} = b.$
$1 \times a - x$	2	$x + \frac{aa - 2ax + xx}{x} = ab - bx,$
$2 \times x$	3	$xx + aa - 2ax + xx = abx - bxx,$
$3 \pm$	4	$2x^2 + bx^2 - 2ax = -aa.$
$4 \div 2 + b$	5	$xx - ax = -\frac{aa}{2+b}.$
$5 \square$	6	$x^2 - ax + \frac{1}{4}a^2 = \frac{1}{4}a^2 - \frac{a^2}{2+b}.$
$6 \sqrt{\phantom{x}}$	7	$x - \frac{1}{2}a = \pm \sqrt{\frac{1}{4}a^2 - \frac{aa}{2+b}}.$
$7 + \frac{1}{2}a$	8	$x = \frac{1}{2}a \pm \sqrt{\frac{1}{4}aa - \frac{aa}{2+b}} = 8273.26 \text{ or } 1725.74,$ the two Numbers.

PROBLEM XXXIII.

ONE lays out 33l. 15s. in Cloths, which he sells again  
at the Rate of 2l. 8s. per Piece, and gain'd as much  
by the whole Bargain as a single Piece cost him; Quere,  
how did he buy in his Cloth?

SOLUTION.

Let 33l. 15s. = 675 Shillings =  $a$ , 2l. 8s. = 48s. =  $b$ ,  
Number of Shillings given for every Piece  $x$ .  
His gain per Piece  $b-x$ .

Proportion,  $x : b-x :: a : \frac{ab-ax}{x}$ , his whole gain,  
which by the Prob. =  $x$ . Then this

Equation

Equation	1	$\frac{ab - ax}{x} = x.$
1 x x	2	$ab - ax = xx.$
2 + ax	3	$xx + ax = ab.$
3 c □	4	$xx + ax + \frac{1}{4}aa = ab + \frac{1}{4}aa.$
4 u u 2	5	$x + \frac{1}{2}a = \sqrt{ab + \frac{1}{4}aa}.$
5 - $\frac{1}{2}a$	6	$x = \sqrt{ab + \frac{1}{4}aa} - \frac{1}{2}a = 45.$

He gave 45 Shillings *per* Piece, and sold them for 48s. *per* Piece, and he bought 15 Pieces, therefore he gain'd 45s. for  $45 : 1 :: 675 : 15,$

Some of the foregoing Problems, with many more such, may be resolved much easier by other Methods *peculiar* to each.

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